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LIFE AND DEATH DECISION ANALYSIS

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DECISION ANALYSIS PROGRAM

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EES

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February 15, 1980

MEMORANDUM TO ADDRESSEES:

Enclosed is a copy of my research report, entitled "Life and Death Decision Analysis". This report is forwarded to your attention in accordance with the designated Distribution List.

Your comments and suggestions on the report are welcome.

Sincerely,

Ronald A. Howard

Ronald A. Howard

Enclosure

ABSTRACT

No assertion can command attention in time of emergency like, "It's a matter of life and death". The problem of making decisions that can affect the likelihood of death is one of the most perplexing facing the analyst. As individuals, we are often called upon to make decisions that affect our safety, and others are increasingly making those decisions on our behalf. Yet most present approaches to life and death-decision making concentrate on the value of an individual's life to others rather than to himself. These approaches are both technically and ethically questionable.

In this report, we develop a model for an individual who wishes to make life and death decisions on his own behalf or who wishes to delegate them to his agents. We show that an individual can use this model if he is willing to trade between the quality and the quantity of his life. A simplified version requires him to establish preference between the resources he disposes during his lifetime and the length of it, to establish probability assessments on these quantities, to characterize his ability to turn present cash into future income, and to specify his risk attitude. We can use this model to determine both what an individual would have to be paid to assume a given risk and what he would pay to avoid a given risk. The risks may range from those that are virtually infinitesimal to those that are imminently life threatening. We show that this model resolves a paradox posed by previously proposed models. In this model there is no inconsistency between an individual's refusing any amount of money, however large, to incur a large enough risk, and yet being willing to pay only a finite amount, his

current wealth, to avoid certain death.

We find that in the normal range of safety decisions, say 10^{-3} or less probability of death, the individual has a small-risk value of life that he may use in the expected value sense for making safety decisions. This small-risk life value applies both to risk increasing and risk decreasing decisions, and is of the order of a few million dollars in the cases we have measured. This small-risk value of life is typically many times the economic value of life that has been computed by other methods. To the extent such economic values are used in decisions affecting the individual, they result in life risks that are in excess of what he would willingly accept. Using the small-risk life value as a basis for compensation should allow most risk-imposing projects to proceed without violating anyone's right to be free from significant involuntarily imposed hazards.

The report demonstrates the use of the model to treat hazards that continue over many years, to determine the size of contributions to saving the lives of others, and to incorporate more precise specifications of consumption-lifetime preferences.

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1. Introduction

No sentence can inspire cooperation in time of emergency more than "it is a matter of life and death". Decision problems whose outcomes could possibly involve death have always been the most perplexing that we face. Yet, formal analysis has provided us with very few tools that can help clarify our choices.

To the decision analyst, death poses one of the most perplexing so-called "intangibles". Diverse outcomes that range from sickness through recreation have been included in a logical analysis, but the evaluation of fatal consequences has seldom proved satisfactory. It seems likely that if we can treat the outcome of death, the acceptability of logical analysis will be significantly increased.

Logical analysis is seen by some as cold and impersonal rather than as warm and human. These people reject immediately a logical analysis of any decision that could have a consequence of death. Of course, some questions are morally tainted, such as, "Would you kill one to save many?", but here the moral problem lies in the decision, not in the method used to make it. We are considering at present not the question of the spiritual nature of human life, but rather the question of how anyone can make decisions that affect

hazards to himself and others in a way that respects everyone's right to freedom from coercion. To act without careful thought and judgment seems less human than following an approach of the type we propose. Indeed, we can find many decisions of a medical or safety nature where using less than the best logic would be irresponsible if not immoral.

The need for a rationale for making decisions involving hazards is evident. Table 1.1 shows a brief list of hazardous decisions--decisions to engage in activities that could be hazardous to the point of death. Neither the list of activities nor the list of the hazards they engender is exhaustive. Many of the activities are clearly enjoyable and pursued in spite of their hazards. We see that the apparently innocuous activity of staying home in bed has its hazards, as well as possibly causing some of the other hazardous activities listed earlier. We note, however, that for virtually all the activities in the table, both the likelihood and consequences of hazard are to some extent controllable by the individual.

Public activities have a corresponding set of hazards. Table 1.2 shows a representative list of public hazardous decisions in terms of the area of the hazard, a typical responsible institution, and examples of specific decisions in each area. We shall not discuss this table in detail, but merely point out that many people are making decisions regarding the safety of others. The question is not whether safety decisions are going to be made, but rather who will make them and how.

Government analysts often use a number like $100,000 or $300,000 for the value of a life.

Table 1.1

Personal Hazardous Decisions

Activity	Sample Hazards
Walking:	DOG ATTACKS, MOTOR VEHICLES, FALLING
Jogging:	DOG ATTACKS, HEART ATTACKS, MOTOR VEHICLES, FALLING
Horseback Riding:	FALLING, BEING KICKED, STRUCK BY BRANCHES
Bicycling:	FALLING, MOTOR VEHICLES
Driving:	MOTOR VEHICLES, ROADSIDE OBSTACLES, FIRES
Motorcycling:	FALLS, MOTOR VEHICLES
Flying:	COLLISION, EQUIPMENT FAILURE
Hang Gliding:	EQUIPMENT FAILURE
Skydiving:	EQUIPMENT FAILURE
Skindiving:	DROWNING
Swimming:	DROWNING
Boating:	DROWNING, FIRES, EXPOSURE
Skiing:	FALLS, COLLISION
Snowmobiling:	ACCIDENTS, FREEZING
Camping:	EXPOSURE, INSECT AND ANIMAL BITES
Traveling:	DISEASE, ACCIDENTS
Sports:	INJURIES, PARALYSIS

EATING:	CHOKING, POISONING, SHORTENED LIFE
DRINKING:	MOTOR VEHICLE ACCIDENTS, CIRRHOSIS OF THE LIVER, SHORTENED LIFE
SMOKING:	CANCER, EMPHYSEMA, SHORTENED LIFE
BATHING:	FALLING, ELECTROCUTION
CONTRACEPTION:	DEATH, ILLNESS
PREGNANCY:	DEATH, ILLNESS
ABORTION:	DEATH, ILLNESS
INNOCULATION:	DEATH
TAKING MEDICINE:	DEATH, ILLNESS
UNDERGOING OPERATIONS:	DEATH, PARALYSIS
STAYING HOME IN BED:	FIRES, BURGLARS, FALLING AIRPLANES, METEORITES, EARTHQUAKES

Table 1.2

PUBLIC HAZARDOUS DECISIONS

AREA	INSTITUTION	EXAMPLE
PRODUCT SAFETY:	CONSUMER PRODUCT SAFETY COMMISSION	POWER MOWERS, TOYS, BICYCLES
WORKPLACE SAFETY:	OCCUPATIONAL SAFETY AND HEALTH ADMINISTRATION	STEPLADDERS
AIRPLANE SAFETY:	FEDERAL AVIATION AGENCY	EVACUATION TRAINING AND EQUIPMENT
HIGHWAY SAFETY:	DEPARTMENT OF TRANSPORTATION	SEAT BELTS, AIR CUSHIONS, BUZZERS, HELMETS
BOATING SAFETY:	COAST GUARD	FLOTATION DEVICE REQUIREMENTS, LIQUEFIED NATURAL GAS SHIPMENT
FOOD SAFETY:	FOOD AND DRUG ADMINISTRATION	CYCLAMATES, SACCHARIN, SODIUM NITRITE
WATER SAFETY:	ENVIRONMENTAL PROTECTION AGENCY	WATER RECYCLING STANDARDS
AIR SAFETY:	ENVIRONMENTAL PROTECTION AGENCY	SULFUR DIOXIDE STANDARDS
DAM SAFETY:	DEPARTMENT OF INTERIOR, CORPS OF ENGINEERS	SITING, CONSTRUCTION, INSPECTION STANDARDS
POWER PLANT SAFETY:	DEPARTMENT OF ENERGY	LIGHT WATER, BREEDER REACTORS
MEDICAL SAFETY:	FOOD AND DRUG ADMINISTRATION, DEPARTMENT OF HEALTH, EDUCATION AND WELFARE	NEW DRUGS, CORONARY CARE UNITS, "UNNECESSARY SURGERY"
ELECTRICAL SAFETY:	UNDERWRITERS' LABORATORIES, LOCAL GOVERNMENT	ALUMINUM WIRE
FIRE SAFETY:	NATIONAL BUREAU OF STANDARDS, DEPARTMENT OF COMMERCE	UPHOLSTERED FURNITURE, CHILDREN'S SLEEPWEAR
METEOROLOGIC SAFETY:	NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION	HURRICANE SEEDING
SPACE RADIATION SAFETY:	NATIONAL ACADEMY OF SCIENCES	FLUOROCARBONS

Perhaps the most common basis for this number is the expected present value of future earnings of the individual. This value may be based on the amount of the individual's earnings flowing to others or to both others and himself. A second basis is the amount of damages juries have awarded in compensation for death. This, in turn, is often at least partially based on calculations of the first type, but it is also tempered by community judgment. A third basis is the amount of life insurance individuals carry. A fourth basis is the computation of the value of life implied by social decisions, or in some cases by individual decisions that have been made in the past. An excellent review of these approaches appears in reference [1].

The difficulty with such approaches is that they all focus on the individual's value to others rather than to himself. If I am making a decision involving risk to myself, I want to use my value to me, not my value to others. Furthermore, if someone else is making a delegated decision on my behalf, I want him to use my value to me, not my value to others. What we need is a comprehensive way to think about the problem of risks involving death on both a private and public basis.

2. Desiderata for a Decision Procedure

Whatever procedure we develop should be able to treat all kinds of risks* involved in both social and private decisions; the voluntary risks that a person imposes on himself as well as the involuntary risks that the government or other members of society impose on him.

While the procedure will be most useful in evaluating low probability-high consequence outcomes, like death, it should treat on a consistent basis all outcomes, even high probability-low consequence outcomes, like perhaps minor eye irritation. And for completeness, it should be able to treat high probability-high consequence decisions, such as the decision to undergo an experimental operation or to play Russian roulette. While we have emphasized death as an outcome, pain, injury, scarring, and days in the hospital should and can be treated [2] [3]. Naturally, the economic consequences of risk, such as economic benefits or property losses, must also be considered.

Whatever procedure is used should be able to treat all sources of risk. In many decisions, the main sources of risk will be accidents. These accidents may be caused by nature, such as earthquakes or hurricanes [4], or by man, such as falling airplanes or ship collisions. The man-made disasters will often be the result of spontaneous failures of systems and safety features, and our procedure must be able to represent these adequately [5]. However, man-made disasters can also result from the unintentional or intentional acts of man. The

*Risk: the possibility of suffering harm or loss

unintentional acts would be those associated with the inattention, incompetence, or incapacity of those responsible for a system's proper operation. The intentional acts could be spiteful actions, sabotage, or war, perhaps the least recognized and least quantified of contemporary risks.

Some decisions may produce continuing risks either instead of or in addition to accidental ones. For example, the decision to build a fossil fuel power plant may pose little risk of accident, but a continuing hazard to health through its particulate and sulfur dioxide emissions. These continuing risks of morbidity and mortality must also be evaluated by the procedure.

3. The Decision Analysis Paradigm

A general procedure of the type we require is provided by decision analysis, a discipline described at length elsewhere [6]. A simple characterization of the decision analysis approach appears in Figure 3.1. A system model is constructed that allows the multi-dimensional set of outcomes to be determined for any setting of the state variables in the problem, the variables that are uncertain and not under the decision-maker's control. When the decision-maker assigns probabilities (indicated by { }) to these state variables, he determines a joint probability distribution on the outcome variables for each alternative. Constructing the system model may not be a simple task - it might require the efforts of many people for several years. But the model and its associated probability assignments capture the information associated with the decision.

The problem then becomes one of determining preference. We divide the assignment of preference into three parts: value assessment, time preference, and risk preference. This division is convenient, but not necessary, and occasionally unachievable. Value assessment refers to trading off one type of outcome for another, such as deaths for injuries, or sound level for money. Time preference requires trading values in the future for values today. In the extreme, this becomes a question of asking what one generation owes the next. Finally, we must assess how expectation is to be traded for surety - the assignment of risk preference. When the information is combined with preference, logic alone determines the best alternative, the

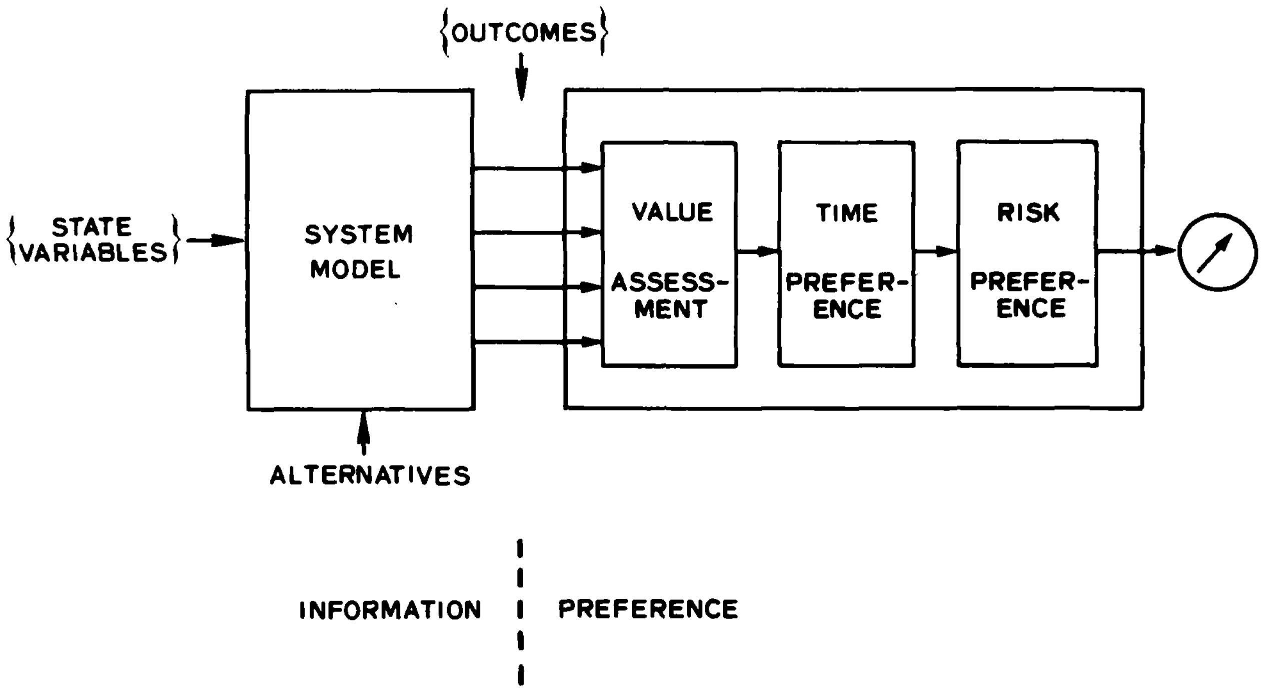


Figure 3.1: The Decision Analysis Paradigm

one that is consistent with the decision-maker's alternatives, information, and preferences.

The main value of the paradigm is not, however, that it finds the best decision, but rather that it provides considerable insight into the problem and a means for communicating about the important issues when several parties are involved. The insight comes from several sources. First, there is the immersion value that any systematic procedure provides for those who follow it. Second, there is the ability to do quantitative sensitivity analysis to determine the relative importance of various features of the problem. Third, there is the unique ability of decision analysis to evaluate what additional information on any uncertain variable would be worth.

The value in communication comes from the use of the language and concepts of decision analysis to facilitate discussion among decision-makers, analysts, and experts in the information areas bearing on the decision. Decision analysis provides a language for describing risky problems that allows us to avoid semantic traps and pointless arguments that do not contribute to the development of understanding.

Application to Hazardous Decisions

The purpose of this work is to develop a methodology for describing preferences in situations where the decision affects the probability of death. We thus concentrate on the preference side of the decision analysis paradigm. Of course, most hazardous decisions will, in addition, require extensive analysis to prepare the information side, to assess the probability of death for each alternative.

4. A Delegable Decision Procedure

Our ethical starting point is that each individual has the right to make or delegate decisions that affect his life. He assigns his preferences on possible outcomes and assesses the probabilities of these outcomes for each possible course of action using his knowledge. He thus bases his decisions on the values and risks he perceives, although others may attempt to persuade him to change his assessments by providing information.

Ideally, every person would make his own life and death decisions. However, there is a large practical advantage if he can delegate some of these decisions to agents in such a way that his rights are respected. We shall explore how the principal can define for the agent both the domain of delegated decisions and how the agent should make a decision within that domain.

To illustrate how we might construct a delegable decision procedure for a decision involving the value of life, let us suppose that I have delegated to an agent the right to make a decision that will affect both net benefit to me and my probability of death. Figure 4.1 describes the situation. There are three alternative system designs: A, B, and C. My annual expected net benefits from the systems, exclusive of the possibility of death, are π_A, π_B, π_C, respectively, with $\pi_A > \pi_B > \pi_C$. The associated annual probabilities of my death that the agent reports and I believe are p_A, p_B, p_C with $p_A > p_B > p_C$. Note that the system with the most desirable economic effect is the one with the highest probability of causing my death.

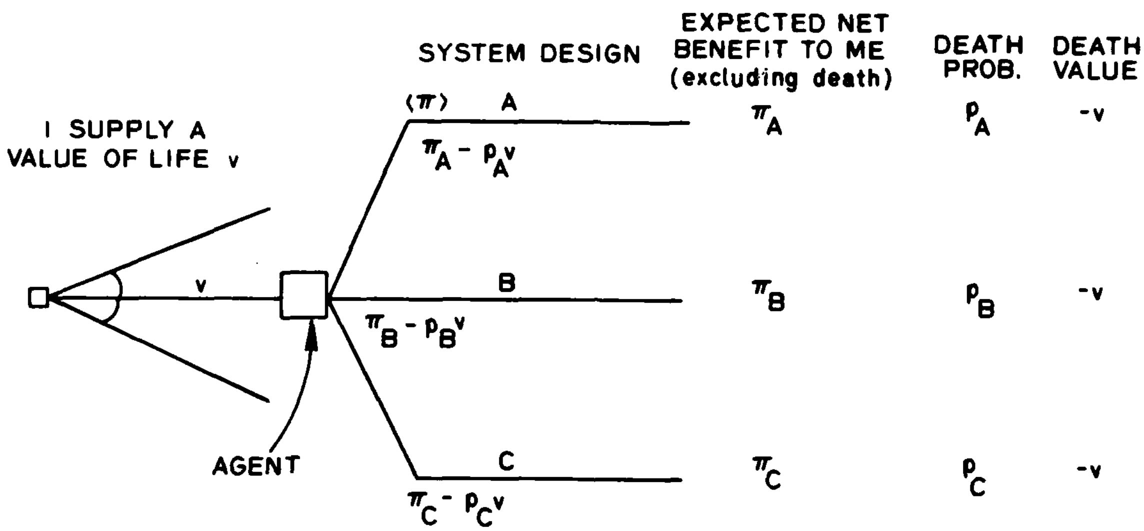


Figure 4.1: A Delegable Decision

Let us suppose that we have the agent act as follows. First, I provide him with a life value v. Then he uses this value to compute $\langle \pi \rangle$, the expected net benefit to me from each of the systems and selects the one for which this quantity is highest. Figure 4.2 shows the effect of my supplying various life values, v. As v increases, the net benefit decreases, but the safety of the resulting system increases. The choice of system moves from A to B to C. In fact, as the dashed lines indicate, in practice there will be a virtual continuum of systems; each v will correspond to a system design with particular economic benefits and safety.

The problem now is what value of v should I provide? If I make it too large, I am very safe, but I receive very little benefit. If I make it too small, I receive a great deal of benefit, but I feel unsafe [5]. Indeed, as we shall see, when p becomes too large, I shall not want the decision made by this simple expected value procedure at all. But we now turn to the question of establishing v for those cases where its use would be appropriate.

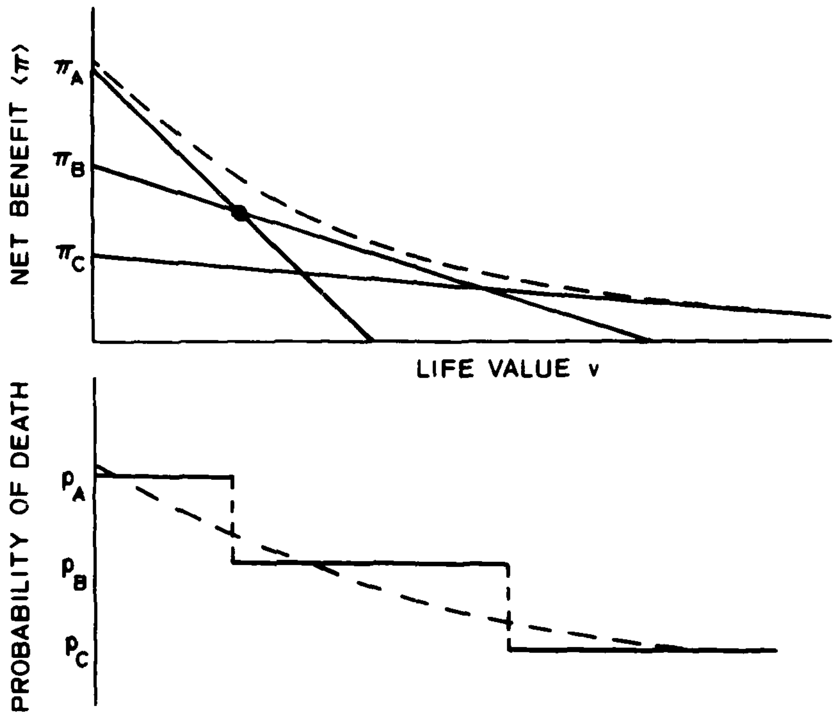


Figure 4.2: Economic and Safety Effects of a Life Value

5. The Issue in the Extreme: The Black Pill

To face this question head-on, let us consider an example I call the black pill. You are offered an opportunity to take a pill which will kill you instantly and painlessly with probability p; you have no doubt about the probability. To induce you to do this, you are offered an amount of money x. For a given p, how large would x have to be before you would be indifferent between taking the pill and not taking it?

Figure 5.1 places the question in a concrete form. Would you accept one chance in 10,000 of death for a payment of $1000? We suppose that an urn has been filled with 10,000 pills, of which one is the indistinguishable, deadly "black pill". A reputable accounting firm assures you that there are, in fact, 10,000 pills in the urn and that there is exactly one deadly pill. Would you swallow a pill from the urn in consideration of a cash payment of $1000?

When I have posed this problem to groups, only a small percentage say that they would accept the proposition. This means that, in the expected value sense, most of the people in the groups are valuing their lives at more than $10,000,000. There is an obvious discrepancy between this number and the value of a few hundred thousand dollars used in certain public decisions. And yet we must question whether people implicitly assigning a value of tens or hundreds of millions of dollars to their lives are being consistent with the levels of resources they command and with the hazards they presently have accepted in work and play. We need some way to think about the black pill proposition in more detail.

WOULD YOU TAKE ONE IN TEN THOUSAND

($\frac{1}{10,000}$) CHANCE OF DEATH FOR $1000

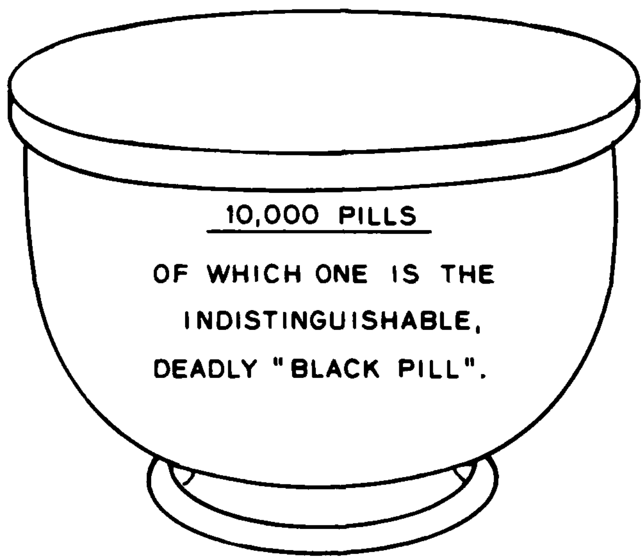


Figure 5.1: The Black Pill

The decision situation is diagrammed in Figure 5.2. Suppose you now have wealth W. If you reject the offer, you will continue your life with wealth W and face whatever future life lottery you presently face. Your future life lottery is the uncertain, dynamic set of prospects you foresee beginning with today. If, on the other hand, you accept the proposition, your wealth will increase to $W + x$. If you live after taking the pill, you will begin your future life lottery with wealth $W + x$, presumably a more desirable situation. If you die, you will leave $W + x$ in your estate, and, of course, have no opportunity to enjoy it. Clearly the value of this estate benefit might be different for different people, and could be included in the analysis. But let us say, for the moment, that it has no value to you. Naturally, there would also be tax effects, but these too we shall ignore.

When I think about this proposition, about what x I would stipulate for different values of p, I arrive at a curve that is qualitatively like that of Figure 5.3. As p increases, I want x to increase. As the values of p become 1/1000 and then 1/100, I demand increasingly large sums of money. In fact, there may be a probability $p = p_{max}$ such that no sum of money, however large, will induce me to play. For example, if $p_{max} < \frac{1}{6}$, then you could never pay me enough to play Russian roulette. The question that remains is how to determine this curve quantitatively from more fundamental preferences.

Before we proceed to this determination, let us discuss other aspects of the black pill situation. First, if you feel that it would

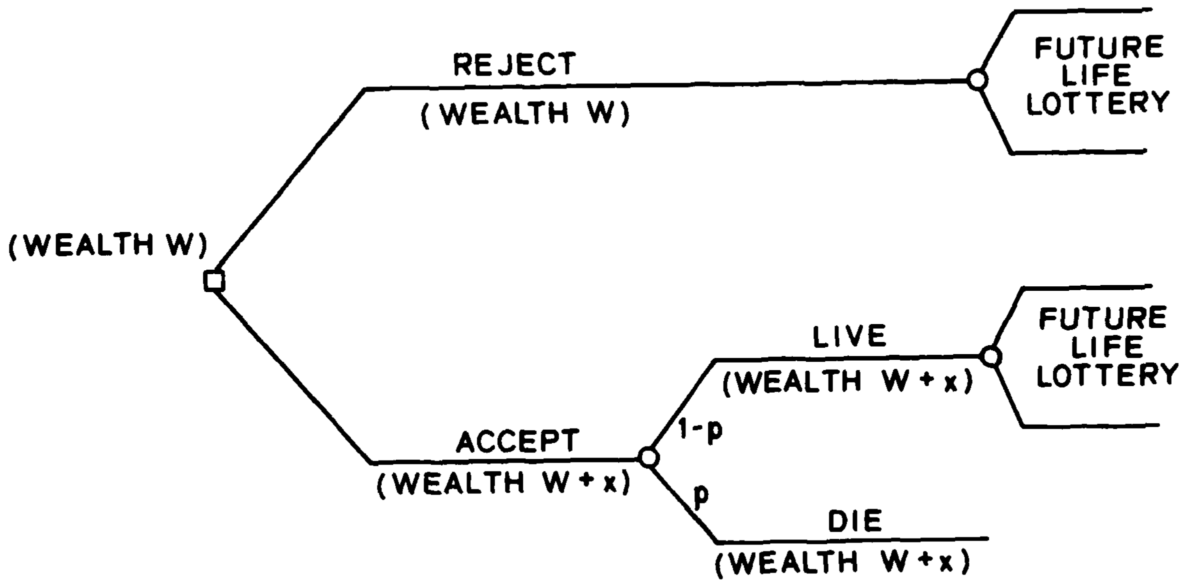


Figure 5.2: The Black Pill Decision Tree

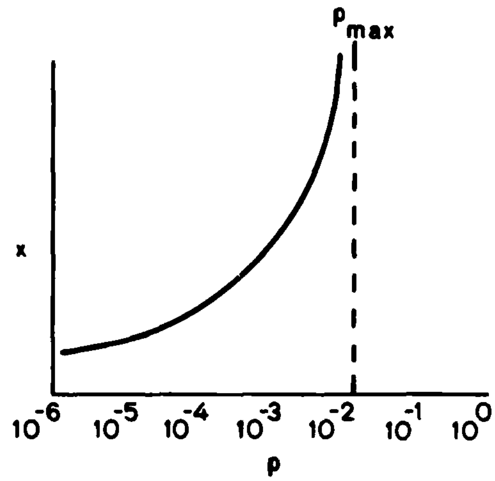


Figure 5.3: The Qualitative Tradeoff

be morally wrong to take such a pill, imagine that the need arises in an experiment that could save many lives or produce other value to mankind. (Consider the plight of the first man who ate a tomato.) We could certainly create just enough moral advantage to taking it to offset your moral objections until the question became the economic one of whether it should be you or someone else who gets the privilege. Every person who travels on business is taking the black pill as part of his work.

There is also the possibility that a person accepting the black pill proposition could use the $1000 he receives to diminish his death risk from other sources by more than 1/10,000. For example, a $1000 medical examination might reduce his death probability by 1/1000.

It may be most helpful to view hazardous decisions as a series of transactions where we are sometimes buying and sometimes selling hazard in our lives for pleasure, money, or other valuable consideration. The question is how to be consistent in such transactions.

To complete our discussion, we must therefore also consider a situation where you face a decrease p in your probability of death and determine how much, x, you would be willing to pay for the decrease. The curve of Figure 5.3 could be extended to include decreases in the probability of death, limited, of course, by the requirement that the absolute probability of death in any time period could not be negative. The amount x that you would pay to reduce the probability of death next year to zero (given that you lived your life as usual) would be the value of a real one-year life "assurance" policy. We consider paying for such decreases in hazard in Section 11.

6. Finding the Value of Life

Our procedure for finding the value of life will be based on the belief that everyone has a fundamental preference on both level of consumption and remaining length of life. By consumption, we mean not only what is literally consumed as opposed to invested, but also the monetary expenditures that contribute to one's standard of living in a general sense, such as charitable contributions. Consumption is thus short for "level of expenditure". We begin by asking an individual how much consumption (measured in today's dollars) he expects to have at each year in the future. For the purpose of constructing a simple preference model, we then ask what constant level of consumption c over his lifetime would make him indifferent between this level and his present prospects. We call this the constant annual consumption for that individual. Now we give him choices between the different futures described by different constant annual consumptions c and different lifetimes, and find to what combinations he is indifferent. The results of this process would appear as in Figure 6.1. Here c is the constant annual consumption and ℓ is the lifetime. Each curve represents the various (c,ℓ) values to which the individual is indifferent; they are indifference curves for the consumption-lifetime choice. Naturally, the individual would prefer to be on an indifference curve that is as far from the origin as possible.

A typical question may help to describe the procedure. We would ask, "Which do you prefer--20 years of life at $10,000 annual consumption or 10 years of life at $30,000 annual consumption, given that you only had these choices?" He might say that he

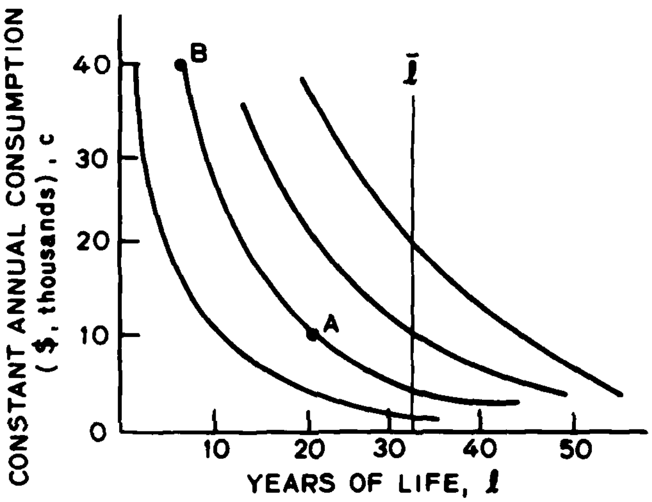


Figure 6.1: Consumption--Lifetime Tradeoffs

preferred 20 years at \$10,000. Therefore, we would increase the consumption involved in the second choice. If we increased the \$30,000 to \$40,000 and he was then indifferent, we would have established the equivalence between (\$10,000, 20 years) and (\$40,000, 10 years) as far as this individual was concerned. This means that points A and B must lie on the same indifference curve, as shown in Figure 6.1.

Each person has a joint probability distribution on annual consumption and lifetime $\{c,\ell\}$ assigned on the basis of his own information or after consultation with experts. This distribution permits computing the expected lifetime $\bar{\ell}$ from the marginal distribution on ℓ. We can use the consumption values corresponding to $\bar{\ell}$ in Figure 6.1 to establish an index, or numeraire, on the utility curves. Thus for any (c,ℓ), we find the $(w,\bar{\ell})$ to which the individual is indifferent and call w the equivalent consumption for expected lifetime that the individual prefers equally to the original (c,ℓ) or simply the worth of the (c,ℓ) pair, $w(c,\ell)$. If we used additional dimensions, like bequests, we could reduce them all to an equivalent w in this way. Thus any certain prospect for consumption and lifetime faced by the individual can be converted into an equivalent consumption for his expected lifetime.

Since the individual's future is, in fact, uncertain and described by $\{c,\ell\}$, we need to measure his risk preference to have a unique way of valuing his future. To illustrate how to do this, suppose that we consider three equivalent consumptions for expected

lifetime w_1, w_2, w_3, that $w_1 > w_2 > w_3$, and that the individual prefers them in that order. Then we would offer him a lottery on w_1 and w_3 with probability q of winning w_1 and ask him for what value of q he would be indifferent between the lottery and receiving w_2 for sure; the choice is diagrammed in Figure 6.2. For example, we might find that he is indifferent between receiving \$20,000 a year for his expected life and participating in a lottery that was equally likely to result in consumptions of \$30,000 or \$15,000 for the same period. Answers to a sufficient number of questions of this type will allow us to complete the individual's preference assignment by measuring his risk preference. The result will be a utility function $u(\cdot)$ on w, and hence on any (c,ℓ) pair.

Suppose now that the individual faces a future life lottery described by $\{c,\ell\}$. The utility to him of this lottery will be

$$\int_{c,\ell} u(w(c,\ell)) \, \{c,\ell\} \ .$$

We can determine $\tilde{w}$, the amount of guaranteed annual consumption for expected lifetime that is equivalent to this lottery by setting the utility of $\tilde{w}$ equal to the utility of the lottery:

$$u(\tilde{w}) = \int_{c,\ell} u(w(c,\ell)) \, \{c,\ell\} \ .$$

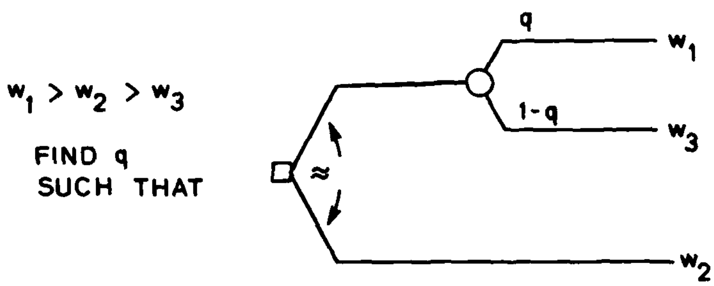


Figure 6.2: Risk Preference Assessment

Suppose that the individual is offered a proposition that could change his level of consumption and his length of life, possibly in a random fashion. For example, suppose he is offered the proposition with the black pill for a specified probability of death and payment (p, x). This will simply change his future life lottery $\{c, \ell\}$ and hence establish a new certain equivalent annual consumption $\tilde{w}\,(p, x)$. To determine the value of x to which he should be indifferent for a given p, we simply adjust x until $\tilde{w}\,(p, x)$ is equal to $\tilde{w}$, the certain equivalent annual consumption he would have in the absence of the proposition. Thus we can use the individual's information and preference to establish the (p, x) pairs to which he is indifferent.

Before proceeding to a discussion of the results of this procedure, we should note that it is readily extendable to include non-uniform consumption, bequests, and taxes. These modifications make computations somewhat more complex but do not change the fundamental nature of the approach.

7. The Form of Results

A possible result of this type of analysis appears in Figure 7.1. The upper curve shows the payment x that the individual would demand to accept a black pill that would kill him with probability p. The payment ranges from less than a dollar when p is less than 10^{-6} to over a million dollars when p is about 1/30. It is entirely possible that the curve will become vertical at some value p_{max} less than one. Risks with $p > p_{max}$ we shall call "unacceptable". Playing Russian roulette may be a risk in this category for most people.

The lower curve shows the value v that the individual would supply to an expected value criterion delegation process. This value is computed by dividing x by p, since the expected value criterion requires that $x = pv$ be the payment for the black pill. This value v will typically become constant for small p and increase as p represents an increasingly more serious risk. Note, as we have indicated, that there may be a value of p such that the individual does not want the expected value procedure used for p greater than this value. We call such risks "undelegable". He is essentially saying to his agent, "When risks are this large, I want to consider the effect of the proposition on my life lottery before arriving at a decision." He would expect major medical operations to fall in this category.

Notice in the upper curve that when p is small enough, the value of x may be only a few dollars. It is likely that for p's

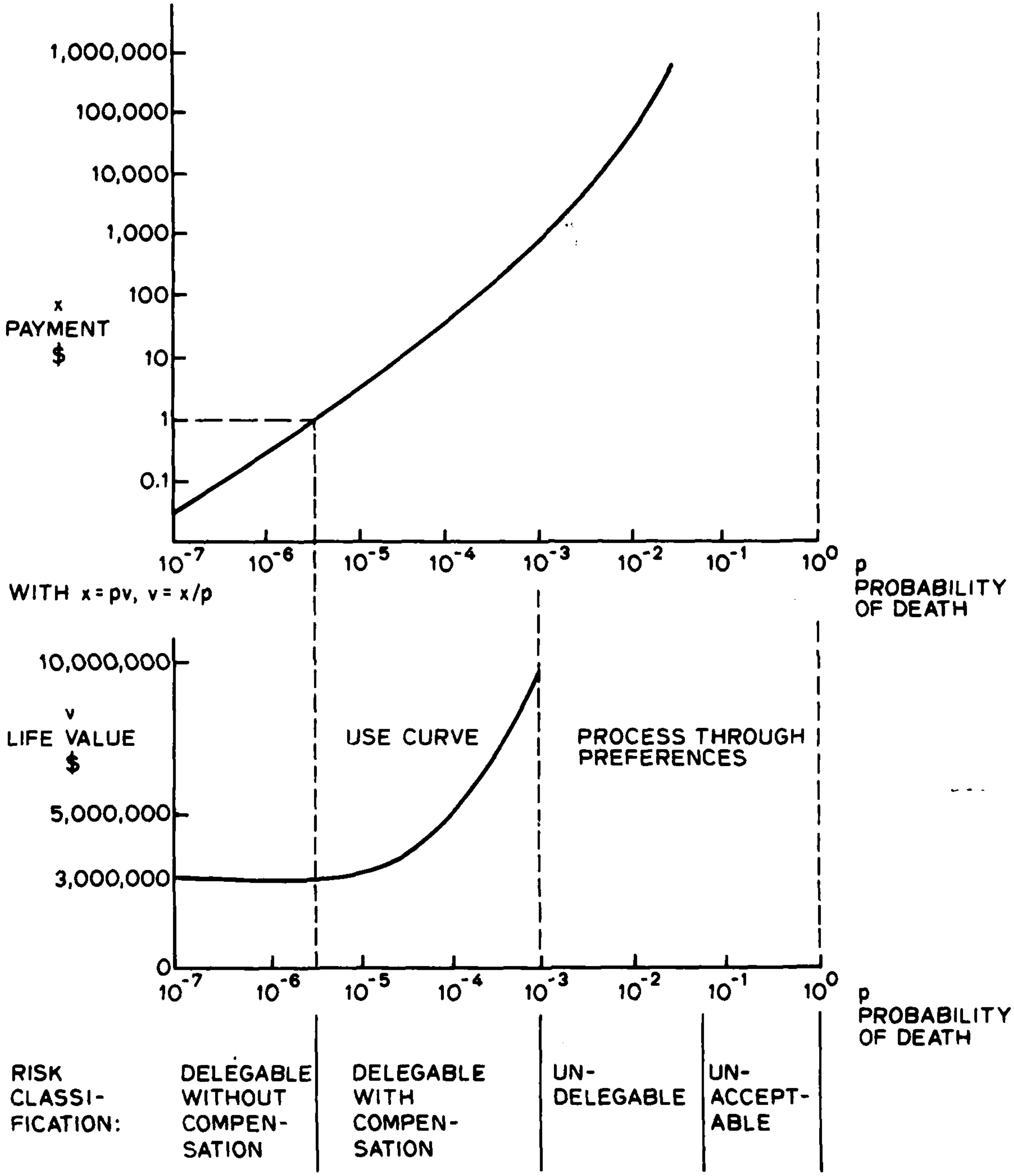


Figure 7.1: Implications of This Type of Analysis

in this range, the individual would be happy to let the agent make the decision without paying the individual any compensation. We call these risks "delegable without compensation". However, when x becomes of the order of tens or hundreds of dollars, the individual would require that the compensation actually be paid. Such risks are "delegable with compensation".

Thus we have specified four ranges of life risk: unacceptable, undelegable, delegable with compensation, and delegable without compensation. The taxonomy is diagrammed in Figure 7.2. An agent making life-risking decisions can only act ethically regarding risks that have been explicitly delegated. The v versus p curve can be very helpful in assuring that the agent is, in fact, properly incorporating the preferences of his principals. Both this curve and the undelegable portion of the x versus p curve may be useful to the individual in arriving at personal decisions; for example, decisions to ski, skydive, or race automobiles.

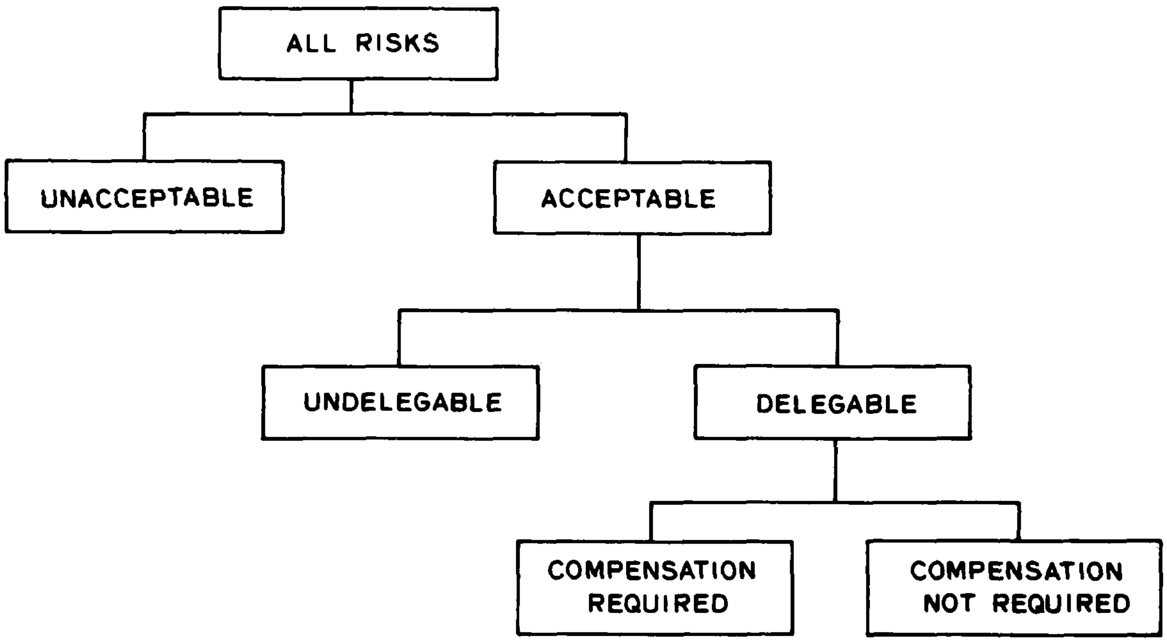


Figure 7.2: A Life Risk Taxonomy

8. A Simple Quantitative Model

To demonstrate the type of analysis proposed, let us simply assume that the individual has some constant annual consumption c that he will receive throughout his life regardless of its length*, and that he is not concerned with bequests. The only question in his mind is then how long he is going to live, ℓ. Thus if he does not accept the black pill proposition, he will receive the utility $u(c,\ell) = u(w(c,\ell))$ with probability $\{\ell\}$ and hence expected utility $\int_{\ell}\{\ell\}\ u(c,\ell)$. This is shown by the upper branch in the decision tree of Figure 8.1. If he accepts the black pill proposition, then he will die with probability p and have neither life nor consumption with utility $u(0,0)$. If on the other hand he accepts the proposition and lives, an event with probability $1-p$, then he receives x, an amount he can use to supplement his consumption. Let us suppose further that if he is given a lump sum x, he will use it to buy an annuity that will allow him to increase his consumption by ζx, for however long he might live. The constant ζ will depend on i, the prevailing interest rate, and on his life distribution $\{\ell\}$ in a way we shall presently discuss. Thus if he lives, he will receive an expected utility $\int_{\ell}\{\ell\}\ u(c + \zeta x,\ell)$.

If the individual is to be indifferent between accepting the proposition and not at the current value of (p,x), then the expected utility of the "accept" and "reject" branches must be the same, and

*The effect of uncertain consumption is explored in Section 14.

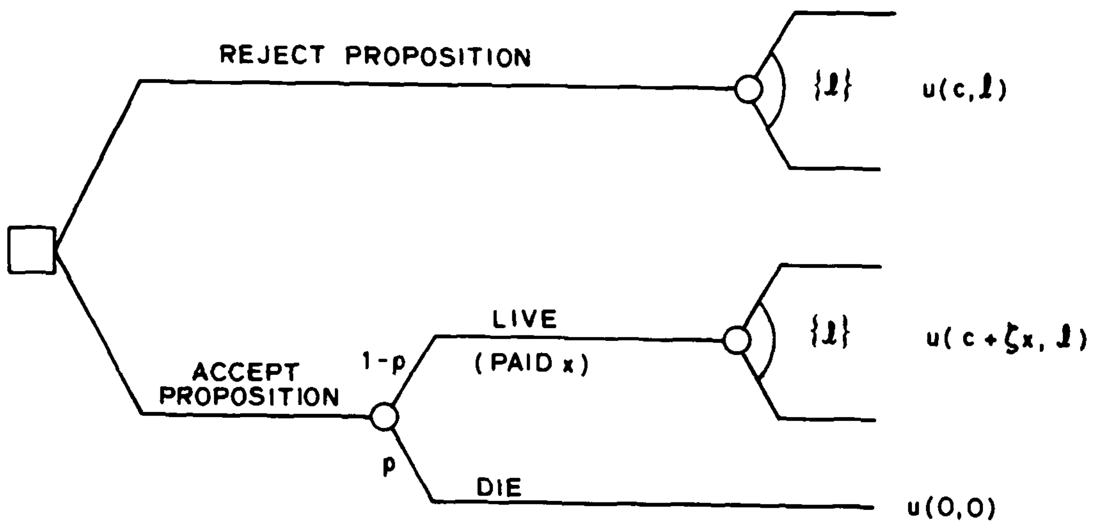


Figure 8.1: A Simple Model

we have,

$$\int_{\ell}\{\ell\}\ u(c,\ell) = p\ u(0,0) + (1-p) \int_{\ell}\{\ell\}\ u(c + \zeta x,\ell)\ . \tag{8.1}$$

Using $\langle\ \rangle$ for the expectation operator, with respect to ℓ in this case, we obtain

$$\langle u(c,\ell)\rangle = p\ u(0,0) + (1-p)\ \langle u(c + \zeta x,\ell)\rangle\ . \tag{8.2}$$

Converting the Payment into Consumption

Before proceeding further in our analysis, let us return to the factor ζ . We assume first, and for simplicity only, that the sellers of annuities use the same distribution on life remaining $\{\ell\}$ that the individual assigns. If the prevailing interest rate* is i, if the individual receives an annual payment ζx beginning today (if he lives), and if he lives ℓ years, then the present value of the payments will be

$$\zeta x \left[1 + \frac{1}{1+i} + \cdots + \left(\frac{1}{1+i}\right)^{\ell-1}\right] \tag{8.3}$$

or

$$\zeta x \frac{1 - \left(\frac{1}{1+i}\right)^{\ell}}{1 - \frac{1}{1+i}} = \zeta x\ \frac{1+i}{i} \left(1 - \left(\frac{1}{1+i}\right)^{\ell}\right).$$

*All payments and interest rates are measured in terms of current dollars, and thus avoid the question of inflation. The analysis could be readily extended to include the individual's beliefs about inflation.

If the seller is indifferent to selling this annuity for x , its expected present value must be x ,

$$x = \zeta x \; \frac{1+i}{i} \; \underset{\ell}{\int} \{\ell\} \left(1 - \left(\frac{1}{1+i}\right)^{\ell}\right)$$

$$= \zeta x \; \frac{1+i}{i} \left(1 - \left\langle \left(\frac{1}{1+i}\right)^{\ell} \right\rangle \right) \quad , \qquad (8.4)$$

or

$$\zeta = \frac{i}{1+i} \quad \frac{1}{1 - \left\langle \left(\frac{1}{1+i}\right)^{\ell} \right\rangle} \quad . \qquad (8.5)$$

Thus, it is a relatively simple matter to compute ζ once i and a lifetime distribution are specified.

These results assume a special form when the interest rate i is zero. First, Expression 8.3 becomes simply $\zeta x \ell$. This, in turn, makes Equation 8.4 read

$$x = \zeta x \bar{\ell} \; , \qquad (8.6)$$

and we have

$$\zeta = \frac{1}{\bar{\ell}} \qquad (\text{when } i = 0). \qquad (8.7)$$

Thus, in the absence of discounting, the amount of lifetime annuity that any investment will buy is obtained by dividing the investment by the individual's expected lifetime.

As the interest rate decreases, the seller of the annuity discounts his future payments less, and hence offers a smaller annuity in return for a given payment: ζ is lower. The interest rate zero corresponds to the lowest value of ζ, the value given by Equation 8.7.

The Maximum Acceptable Probability of Death, p_{max}

Now we obtain the benefits of our work. First, by solving for p in Equation 8.2,

$$p = \frac{\langle u(c,\ell)\rangle - \langle u(c + \zeta x,\ell)\rangle}{u(0,0) - \langle u(c + \zeta x,\ell)\rangle} \quad , \tag{8.8}$$

we obtain an equation that will allow us to find the p corresponding to any x once the utility function is specified. Since $p \to p_{max}$ as $x \to \infty$, we have

$$p_{max} = \frac{\langle u(c,\ell)\rangle - \langle u(\infty,\ell)\rangle}{u(0,0) - \langle u(\infty,\ell)\rangle} = \frac{\langle u(\infty,\ell)\rangle - \langle u(c,\ell)\rangle}{\langle u(\infty,\ell)\rangle - u(0,0)} \tag{8.9}$$

as the equation that will determine p_{max} from the specified utility function.

The Small-Risk Life Value v_s

Since we may expect an individual (a) to prefer a life with unlimited consumption at least as much as the future life lottery he currently faces, and also (b) to prefer his current situation at least as much as death, we have

$$\langle u(\infty,\ell)\rangle \geq \langle u(c,\ell)\rangle \geq u(0,0) .$$

This means that unless equality holds throughout, the maximum acceptable probability of death p_{max} must lie between zero and one,

$$0 \leq p_{max} \leq 1 .$$

If condition (a) is a strict preference, as we would generally expect, then $\langle u(\infty,\ell)\rangle > \langle u(c,\ell)\rangle$ and $p_{max} > 0$. This means that there will be some life risks so small that the individual can be induced to undertake them for money. If condition (b) is a strict preference, as we would generally expect, and if $\langle u(\infty,\ell)\rangle$ is not infinite (there is no consumption so large that the individual prefers any non-zero probability of achieving it, however small, to his present prospects), then $\langle u(c,\ell)\rangle > u(0,0)$ and $p_{max} < 1$. This means that there will be some life risks so large that the individual cannot be induced to undertake them for money.

Consequently, virtually no one can say that he will not risk his life for money or that he will accept any risk for enough money.

Let us now investigate the situation where the probability of death is small. If $x(p)$ is the minimum payment required to accept a death probability p and if $v(p)$ is the value of life that would lead to this payment in an expected value sense, then

$$x(p) = pv(p)$$

or

$$v(p) = \frac{x(p)}{p} \quad .$$

When x is small, we expect $v(p)$ to approach some small risk value of life v_s under conditions that we shall soon explore,

$$v_s = \lim_{p \to 0} v(p) \quad .$$

Then we have

$$v_s = \lim_{p \to 0} v(p) = \lim_{p \to 0} \frac{x(p)}{p} \quad .$$

This limit is indeterminate since $x(0) = 0$. However, by l'Hôpital's rule,

$$v_s = \left.\frac{dx}{dp}\right|_{\substack{p=0\\x=0}} = \frac{1}{\left.\frac{dp}{dx}\right|_{\substack{p=0\\x=0}}} \quad . \tag{8.10}$$

We now differentiate Equation 8.2 with respect to x,

$$0 = \frac{dp}{dx}\; u(0,0) - \frac{dp}{dx}\left\langle u(c+\zeta x, \ell)\right\rangle + (1-p)\cdot\zeta\left\langle \frac{\partial}{\partial c} u(c+\zeta x, \ell)\right\rangle .$$

With $x = 0$, $p = 0$ we have,

$$0 = \left.\frac{dp}{dx}\right|_{\substack{p=0\\ x=0}} \Big(u(0,0) - \left\langle u(c,\ell)\right\rangle\Big) + \zeta\left\langle \frac{\partial}{\partial c} u(c,\ell)\right\rangle$$

or

$$v_s = \frac{1}{\left.\frac{dp}{dx}\right|_{\substack{p=0\\ x=0}}} = \frac{\left\langle u(c,\ell)\right\rangle - u(0,0)}{\zeta\left\langle \frac{\partial}{\partial c} u(c,\ell)\right\rangle} . \tag{8.11}$$

This equation allows us to compute the small-risk value of life directly from the utility function, c, ζ, and the lifetime distribution.

We can provide some insight into this result by developing it in another way. If an individual faced a small black-pill probability p of death and had a small-risk life value v_s, then he would ask $v_s p$ to assume the risk. He would then turn this payment into annual consumption increases of $v_s p\zeta$. This increase in annual consumption would increase his utility by

$$v_s p\zeta \left\langle \frac{\partial}{\partial c} u(c,\ell)\right\rangle .$$

On the other hand, his utility from the prospect of taking the pill is

$$(1-p)\left\langle u(c,\ell)\right\rangle + pu(0,0)$$

rather than his present utility of $\langle u(c,\ell)\rangle$. The difference is

$$-p\ \left[\langle u(c,\ell)\rangle - u(0,0)\right] .$$

The utility increase from the increased consumption plus the utility decrease from the death risk must net to zero if he is to be indifferent:

$$v_s p\zeta \left\langle \frac{\partial}{\partial c}\, u(c,\ell)\right\rangle - p\ \left[\langle u(c,\ell)\rangle - u(0,0)\right] = 0 .$$

If we now divide by p and solve for v_s , we obtain Equation 8.11. We see that the small-risk value will increase as the utility of the individual's future prospects $\langle u(c,\ell)\rangle$ is large compared to his utility of death $u(0,0)$, and as his expected utility of a unit of additional consumption

$$\left\langle \frac{\partial}{\partial c}\, u(c,\ell)\right\rangle$$

decreases.

Note, too, that the small-risk value of life v_s is inversely proportional to ζ . As the interest rate i decreases, ζ decreases, and hence v_s increases. An interest rate of zero would thus produce the highest value of v_s . This result makes sense because as the interest rate decreases the individual will receive less future consumption in return for any payment and so he will require a larger payment before agreeing to a given risk.

We can examine Equation 8.11 to determine when the small-risk value of life will exist. First, we note that the numerator will be positive when the individual prefers his present situation to certain death, the strict form of preference (b) above. Second, we observe that $\frac{\partial}{\partial c} u(c,\ell)$ will be non-negative if we reasonably assume that the individual's satisfaction will not be decreased by additional consumption in the sense we have defined it. Furthermore, if $\frac{\partial}{\partial c} u(c,\ell)$ is positive for at least one (c,ℓ) pair that has positive probability, then the denominator of the v_s expression will be positive. This will happen as long as it is possible for the individual to encounter a life state where increased consumption would bring increased satisfaction, a condition that will usually be the case. Thus, in almost every practical situation v_s will exist. The question of how small p must be before $v_s(p)$ is well-approximated by v_s depends upon the specific utility function selected.

The Worth Function

Now let us become specific about the utility function, $u(c,\ell) = u(w(c,\ell))$. For the worth function specifying indifference curves between c and ℓ, we shall assume the form

$$w(c,\ell) = c \left(\frac{\ell}{\bar{\ell}}\right)^{\eta} \quad \eta > 0 \quad . \tag{8.12}$$

Here $\bar{\ell}$ is the expected lifetime remaining at the time the worth function is assessed. We include it simply to have a convenient numeraire. There is no implication that the worth function will change as expected lifetime changes due to the passage of time or other changes in the life lottery. Thus, at the time of the assessment, we can equivalently think of ℓ as specifying the year of death and $\bar{\ell}$ as the expected year of death.

Suppose that an individual were assured to live his life expectancy. Then $\eta = 2$, for example, would mean that if we cut his life in half, we would have to quadruple his consumption to make him indifferent to the change. Higher powers of η would require greater increases in consumption.

Risk Preference: Exponential

For risk preference $u(w)$ we shall use the exponential form

$$u(w) = -e^{-\gamma w} = -e^{-w/\rho} \tag{8.13}$$

where γ is the risk aversion coefficient and ρ, its reciprocal, is the risk tolerance, in this case as specified for lotteries on worth, or equivalent annual consumption. Then we have

$$u(c,\ell) = u(w(c,\ell)) = -e^{-\gamma c \left(\frac{\ell}{\bar{\ell}}\right)^{\eta}} \tag{8.14}$$

and

$$\frac{\partial}{\partial c} u(c,\ell) = \gamma \left(\frac{\ell}{\bar{\ell}}\right)^{\eta} e^{-\gamma c \left(\frac{\ell}{\bar{\ell}}\right)^{\eta}} . \tag{8.15}$$

With these specifications, Equation 8.8 becomes

$$p = \frac{\left\langle e^{-\gamma c\left(\frac{\ell}{\bar{\ell}}\right)^{\eta}}\right\rangle - \left\langle e^{-\gamma(c + \zeta x)\left(\frac{\ell}{\bar{\ell}}\right)^{\eta}}\right\rangle}{1 - \left\langle e^{-\gamma(c + \zeta x)\left(\frac{\ell}{\bar{\ell}}\right)^{\eta}}\right\rangle} \quad . \tag{8.16}$$

One interesting observation at this point is that as $x \to \infty$, $p \to p_{max}$ as given by

$$p_{max} = \left\langle e^{-\gamma c\left(\frac{\ell}{\bar{\ell}}\right)^{\eta}}\right\rangle \tag{8.17}$$

which must, of course, be less than one. By referring to Equation 8.14, we note that p_{max} for the exponential utility function we have chosen is just the negative of the expected utility of the future life lottery. Thus, we have obtained not only an equation to use in determining the x versus p curve, but also a way to compute p_{max} for this form of utility function.

With the same specifications, the small-risk value of life from Equation 8.11 becomes

$$v_s = \frac{1 - \left\langle e^{-\gamma c\left(\frac{\ell}{\bar{\ell}}\right)^{\eta}}\right\rangle}{\zeta\gamma \left\langle \left(\frac{\ell}{\bar{\ell}}\right)^{\eta} e^{-\gamma c\left(\frac{\ell}{\bar{\ell}}\right)^{\eta}}\right\rangle} \tag{8.18}$$

and we can determine v_s directly when appropriate.

The Economic Life Value, v_e

For purposes of comparison, it is interesting to compute an "economic" value of life v_e based on the expected discounted consumption. An annual consumption c would have an expected present value v_e determined from

$$c = \zeta v_e \tag{8.19}$$

where ζ is defined by Equation 8.5. Thus,

$$v_e = \frac{c}{\zeta} = c\nu \tag{8.20}$$

where

$$\nu = \frac{1}{\zeta} \tag{8.21}$$

is the number of years of consumption that would equal the economic value, a frequently useful quantity.

Note that when $i = 0$, $\zeta = \frac{1}{\bar{\ell}}$ as given by Equation 8.7. Hence in this case

$$\nu = \bar{\ell} \qquad (i = 0) \tag{8.22}$$

and

$$v_e = c\bar{\ell} \qquad (i = 0) \quad . \tag{8.23}$$

When the interest rate is zero, the economic value of life becomes simply the annual consumption multiplied by the expected lifetime.

The Case of Risk Neutrality

We can determine very easily the special forms these results assume for an individual who is risk neutral for lotteries on equivalent annual consumption. This means that the individual has a utility function $u(c,\ell)$ that is linear in $w(c,\ell)$ (and hence $\langle u(\infty,\ell)\rangle$ is infinite). For such an individual γ approaches zero, and we see immediately from Equation 8.17 that $p_{max} = 1$: the individual will accept any p for a large enough x. As γ approaches zero, Equation 8.16 becomes approximately

$$p \approx \frac{\left\langle 1 - \gamma c \left(\frac{\ell}{\bar{\ell}}\right)^{\eta} \right\rangle - \left\langle 1 - \gamma(c + \xi x) \left(\frac{\ell}{\bar{\ell}}\right)^{\eta} \right\rangle}{1 - \left\langle 1 - \gamma(c + \xi x) \left(\frac{\ell}{\bar{\ell}}\right)^{\eta} \right\rangle}$$

$$\approx \frac{\gamma \xi x \left\langle \left(\frac{\ell}{\bar{\ell}}\right)^{\eta} \right\rangle}{\gamma (c + \xi x) \left\langle \left(\frac{\ell}{\bar{\ell}}\right)^{\eta} \right\rangle}$$

$$\approx \frac{\xi x}{c + \xi x} \tag{8.24}$$

with the final expression holding exactly for $\gamma = 0$. By solving for x we obtain

$$x = \frac{c}{\xi} \frac{p}{1-p} \quad . \tag{8.25}$$

$$= v_e \frac{p}{1-p}$$

When the individual is risk indifferent, the required payment x for a given p is just the economic value of life multiplied by $\frac{p}{1-p}$, the odds of dying. The payment becomes infinite as p approaches one. This same expression shows that the small-risk value v_s is just the economic value v_e, a result confirmed by finding the limiting form of Equation 8.18 as γ approaches zero.

The Case of Certain Lifetime

As a final special case, consider the interesting, but highly improbable, situation where the individual believes that he will live his expected remaining life $\bar{\ell}$, no more, no less. Then we find p_{max} from Equation 8.17,

$$p_{max} = e^{-\gamma c} \tag{8.26}$$

and the small-risk value from Equation 8.18,

$$v_s = \frac{1-e^{-\gamma c}}{\zeta\gamma e^{-\gamma c}} = \frac{e^{\gamma c}-1}{\zeta\gamma}\ , \tag{8.27}$$

where

$$\zeta = \frac{i}{1+i} \cdot \frac{1}{1-\left(\frac{1}{1+i}\right)^{\bar{\ell}}} \tag{8.28}$$

from Equation 8.5.

9. The Assessment Procedure

Specifying the model requires assigning values to the remaining life distribution, the interest rate i, the equivalent annual consumption c, the risk tolerance $\rho = \frac{1}{\gamma}$, and the consumption-lifetime tradeoff parameter η.

The lifetime distribution $\{\ell\}$ can be directly assessed or more commonly based upon mortality tables. The type we shall use appears in Table 9.1 [7]. The corresponding probability mass function appears as Figure 9.1. If we use the mortality table, we need only know the person's age in order to truncate and renormalize it appropriately. We shall assume that our "base-case" individual is a male of age 25. This will give him an expected remaining life of 46.2 years. We shall deal in before-tax inflation-free dollars and let the interest rate and the annual consumption be assessed in these terms. We assume that the base-case individual believes that the interest rate is 5% ($i = 0.05$) and that his equivalent annual consumption will be $c = \$20{,}000$.

Now we must assess risk preference. We shall use two methods. The first will be to offer the individual a lottery on annual lifetime consumption that will increase it by y and decrease it by $y/2$ with equal probability. For example, with $y = \$2000$, the lottery would be able equally likely to give him a lifetime consumption of \$22,000 or \$19,000. If he likes the lottery, we increase y until he is just indifferent between accepting it and not. Suppose that for the base-case individual this happens when $y = \$6000$: he is just indifferent between his present situation and a lottery that will provide him for life with an annual consumption of \$26,000 or \$17,000 with equal probability. At this point, we would say that his risk tolerance ρ is approximately \$6000.

Table 9.1

Life Table for White Males, U.S.
of 100,000 Born Alive, Number Dying During Age Interval

Age	Number Dying During Age Interval	Age	Number Dying During Age Interval	Age	Number Dying During Age Interval	Age	Number Dying During Age Interval
0	2592	28	137	56	1295	84	2280
1	149	29	141	57	1383	85	2096
2	99	30	147	58	1486	86	1898
3	78	31	154	59	1598	87	1693
4	67	32	161	60	1714	88	1490
5	60	33	170	61	1827	89	1288
6	55	34	180	62	1935	90	1086
7	52	35	194	63	2039	91	888
8	47	36	210	64	2136	92	709
9	43	37	229	65	2231	93	548
10	40	38	251	66	2323	94	413
11	40	39	278	67	2409	95	300
12	46	40	306	68	2487	96	216
13	56	41	339	69	2559	97	152
14	73	42	376	70	2621	98	103
15	90	43	415	71	2678	99	70
16	107	44	458	72	2729	100	45
17	121	45	505	73	2775	101	29
18	134	46	556	74	2815	102	17
19	143	47	613	75	2841	103	11
20	153	48	681	76	2853	104	6
21	162	49	754	77	2855	105	3
22	167	50	835	78	2844	106	2
23	163	51	916	79	2821	107	1
24	157	52	995	80	2789	108	1
25	149	53	1071	81	2738		
26	141	54	1144	82	2639		
27	137	55	1216	83	2482		

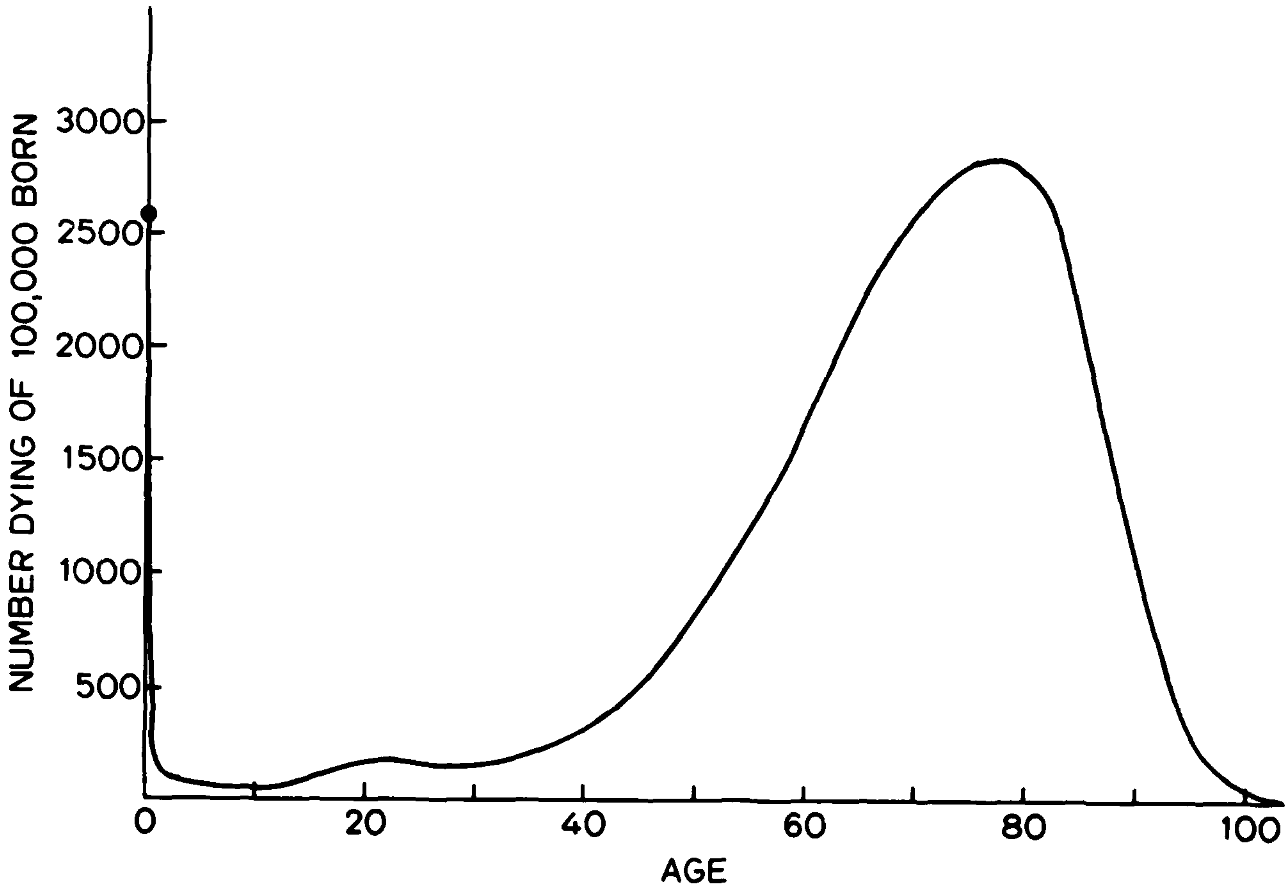


Figure 9.1: Lifetime Mass Function

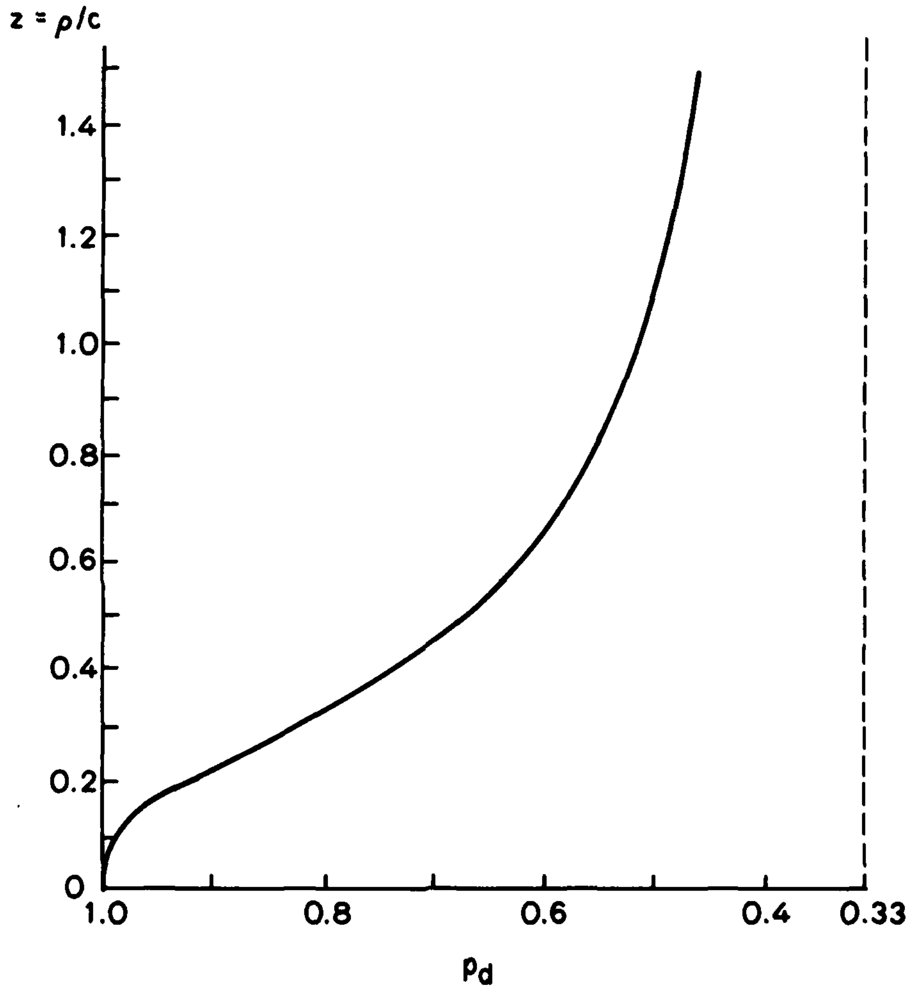
A second way to assess ρ is to ask for what probability p_d of doubling his consumption to $2c$ and $1-p_d$ of halving it to $c/2$ he would be just indifferent to receiving c, in each case for life. A value of p_d of 0.82 would also correspond to a ρ of \$6000. That is, someone with a ρ of \$6000 would be just indifferent to 4.5 to one odds of doubling or halving his consumption. Naturally, as p_d increases, the risk tolerance decreases. The relation is

$$p_d = \frac{e^{-\frac{1}{2z}} - 1}{e^{-\frac{3}{2z}} - 1} \quad \text{where } z = \rho/c \ . \qquad (9.1)$$

This relation between p_d and ρ/c appears as Figure 9.2. As ρ/c and hence z becomes unboundedly large, the individual should make decisions on an expected value basis. The limit of p_d as z approaches infinity is readily found to be 1/3. With $p_d = 1/3$, the expected consumption to be obtained from the lottery is just the present consumption, c.

If these two procedures provide different values of ρ, one can ask the individual with respect to which answer he is more comfortable and then adjust the other until a reasonable value is obtained.

The last task is to assess η. Suppose that an individual faces the prospect of receiving annual consumption c for his expected lifetime $\bar{\ell}$. The question is then if we change his lifetime to $\beta\bar{\ell}$, what new annual consumption αc would just make him indifferent to the original arrangement. That is



Determining ρ from p_d

Figure 9.2

$$w(c,\bar{\ell}) = w(\alpha c, \beta\bar{\ell}) \quad ,$$

$$c\left(\frac{\ell}{\bar{\ell}}\right)^{\eta} = \alpha c \left(\frac{\beta\ell}{\bar{\ell}}\right)^{\eta} \quad ,$$

$$1 = \alpha\beta^{\eta} \quad .$$

or

$$\eta = -\frac{\ln \alpha}{\ln \beta} \quad . \tag{9.2}$$

This question can be asked for β both greater and less than one and the results compromised, if necessary.

Suppose, for example, that when we faced the individual with a 5% reduction in life $(\beta = 0.95)$, he responded that he would need a 10% increase in consumption $(\alpha = 1.1)$ to make him equally happy. Then

$$\eta = -\frac{\ln 1.1}{\ln 0.95} = 1.86 \; .$$

For our base-case individual, we shall use $\eta = 2$. This means, as we have said, that halving the lifetime requires quadrupling the consumption $(\alpha = 4, \beta = 0.5)$. A fairly complete set of tradeoff curves appears in Figure 9.3.

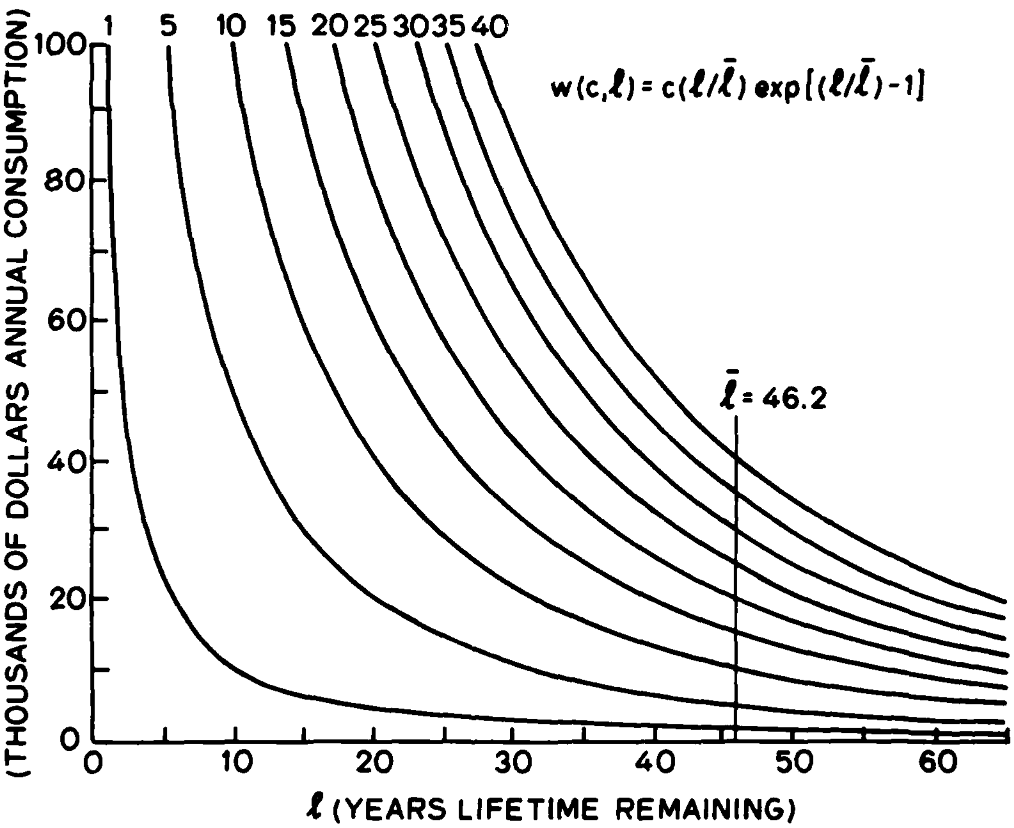


Figure 9.3: The Consumption Lifetime Tradeoff Function

10. Numerical Results*

The mortality table plus age 25 specifies $\{\ell\}$, and we have determined $c = \$20{,}000$, $\rho = \$6{,}000$, $i = 0.05$, $\eta = 2$. Thus we have all the information necessary to exercise the model. The results appear in Figure 10.1. The small-risk value of life v_s is \$2.43 million dollars. The value of life grows to \$100 million dollars when p is about 0.1. No payment will induce the individual to assume a risk of dying greater than $p_{max} = 0.103$. This individual would not play Russian roulette under any circumstances. If we define the range of delegable decisions to be those for which v is approximately v_s, we see that this individual might be willing to delegate for p as large as 10^{-2}. A compensation of at least \$10 would be required whenever p exceeded 4×10^{-6}.

These results determine the answer of the base case individual to the black pill problem posed in Figure 5.1. With $v_s = \$2.43$ million and $p = \frac{1}{10{,}000}$, the compensation x would be approximately $v_s p = \$243$, as indicated by the upper curve of Figure 10.1. Since the \$1000 offered for taking the pill exceeds \$243, the individual would be consistent in accepting the black pill proposition.

The economic value v_e is \$363,000, obtained from $c = \$20{,}000$ and $\nu = 18.15$. This value is roughly comparable to those based on traditional economic approaches. Note that the small risk value v_s is about 6.7 times the economic value. If this model and the numbers

*The computations in Sections 10 and 11 were originally performed by Dr. Mary D. Schrot.

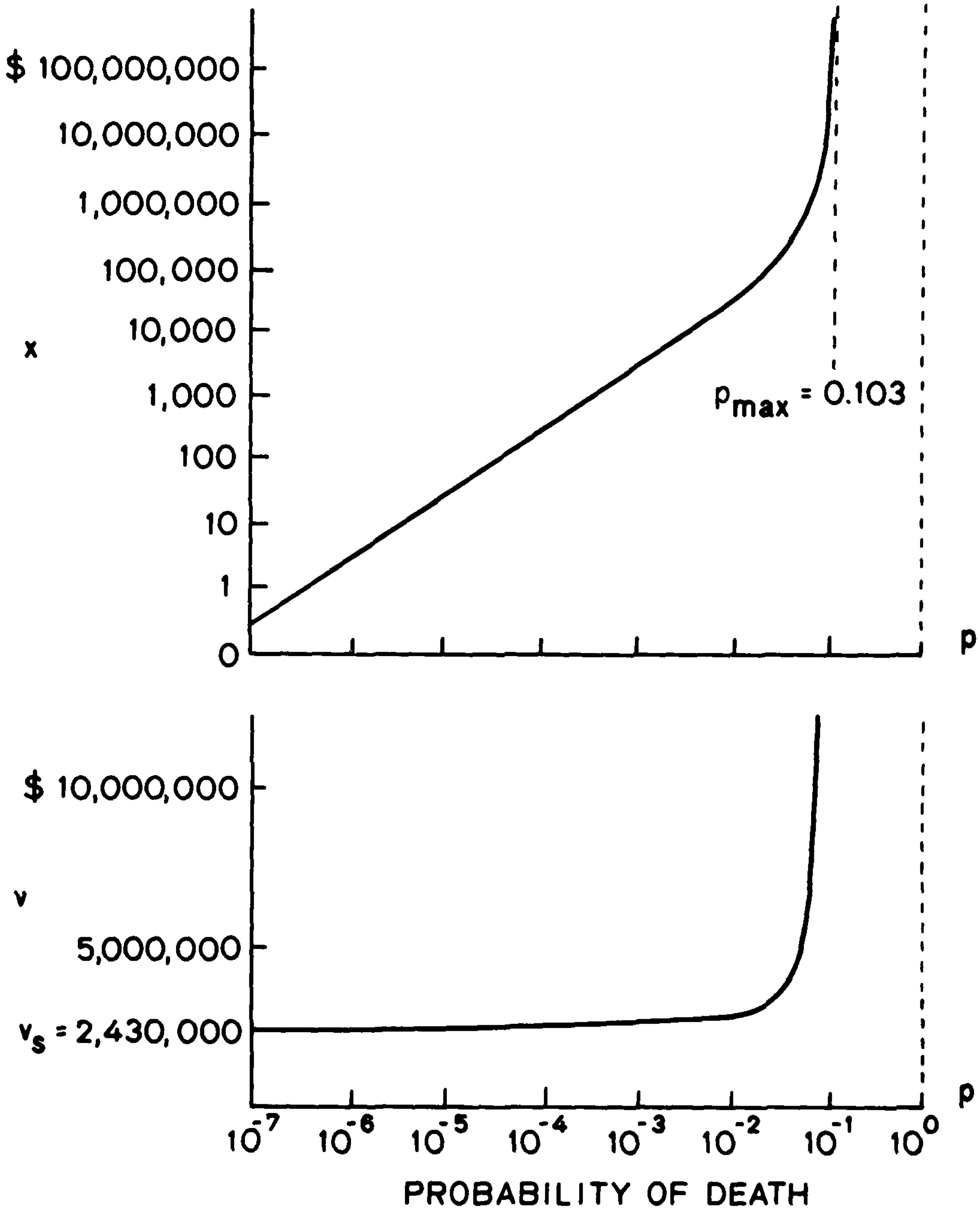


Figure 10.1: Black Pill Results

used in it are representative, the economic values that have been used in past governmental decision making would considerably underestimate the individual's own value.

Table 10.1 shows how the results depend on each of the model parameters. The first row of lines shows the effect of changing the base case by varying consumption from \$10,000 to \$30,000 per year. The first line shows that the small risk value v_s will vary from \$0.529 million to \$6.648 million, a very large change. The next line shows that the economic value will vary from \$182,000 to \$545,000. The economic value is in direct proportion to consumption, as required. The third line displays the ratio of these quantities, which exhibits a wide variation from 2.91 to 12.20. Finally, the right side of the table shows the effect on p_{max}. When consumption is \$10,000, p_{max} becomes as high as 0.24; when consumption is \$30,000, p_{max} falls to 0.0611. Thus, when consumption is increased, but risk tolerance and the other parameters remain fixed, the individual becomes less willing to take large chances with his life.

The second row shows the effect of varying risk tolerance from \$10,000 to \$3000. This has the effect of sweeping the small-risk value of life from \$1.277 million to \$6.903 million. The economic value does not change since it does not depend on ρ. This range of risk tolerance variation causes p_{max} to fall from 0.195 to 0.0431, as we would expect.

Table 10.1

Sensitivity Analysis

Base case: $c = 20{,}000$ $\rho = 6{,}000$
$i = 0.05$ $\eta = 2$
Age 25

		Nominal			v_s (\$ million) v_e (\$ million) v_s/v_e Nominal			p_{max} Nominal	
c:	10,000	20,000	30,000	0.529	2.430	6.648	0.240	0.103	0.0611
				0.182	0.363	0.545			
				2.91	6.69	12.20			
ρ:	10,000	6,000	3,000	1.277	2.430	6.903	0.195	0.103	0.0431
				0.363	0.363	0.363			
				3.52	6.69	19.02			
c:	10,000	20,000	30,000	1.215	2.430	3.645	0.103	0.103	0.103
ρ:	3,000	6,000	9,000	0.182	0.363	0.545			
				6.69	6.69	6.69			
i:	0.10	0.05	0.025	1.421	2.430	3.622	0.103	0.103	0.103
				0.212	0.363	0.541			
				6.70	6.69	6.70			
η:	3	2	1	2.418	2.430	2.541	0.140	0.103	0.0640
				0.363	0.363	0.363			
				6.66	6.69	7.0			
Age:	35	25	15	2.157	2.430	2.671	0.125	0.103	0.0920
				0.334	0.363	0.381			
				6.46	6.69	7.01			

However, varying risk tolerance from \$10,000 to \$3,000 for a person with \$20,000 annual consumption is quite extreme. For such a person, a risk tolerance of \$10,000 corresponds to a doubling probability p_d of 0.67, whereas a \$3,000 risk tolerance implies a p_d of 0.96. These values are considerably different from the 0.82 value for p_d that we have assumed. A value of p_d near 0.8 seems to be most often favored among the individuals I have interviewed.

Another interesting analysis is to vary c, but to maintain c/ρ constant. This is done in the third row of the table. We observe that the small-risk value varies from \$1.215 million to \$3.645 million. However, the small-risk value remains proportional to the economic value. The reason is apparent when we divide Equation 8.18 by Equation 8.20.

$$\frac{v_s}{v_e} = \frac{1 - \left\langle e^{-\gamma c \left(\frac{\ell}{\bar{\ell}}\right)^{\eta}} \right\rangle}{\gamma c \left\langle \left(\frac{\ell}{\bar{\ell}}\right)^{\eta} e^{-\gamma c \left(\frac{\ell}{\bar{\ell}}\right)^{\eta}} \right\rangle} . \qquad (10.1)$$

The ratio v_s/v_e depends on c and ρ only through the ratio $c/\rho = \gamma c$.

We also note that p_{max} remains at 0.103 throughout the variation. Equation 8.17 shows that p_{max} also depends on c and ρ only through c/ρ. *Ceteris paribus,* if an individual changes his risk tolerance in proportion to his consumption, his p_{max} will remain unchanged according to this model.

Row four shows the effect of varying the interest rate from 0.10 to 0.025. The small-risk value v_s grows from \$1.421 million to \$3.622 million as the interest rate decreases: low interest rates lead to high small-risk life values because the individual requires a higher payment to yield the same consumption increase. Note, too, that v_s is proportional to the economic value v_e. Equation 10.1 confirms the appropriateness of this result by revealing that the ratio v_s/v_e depends in no way on the interest rate. Equation 8.17 likewise reveals that p_{max} does not depend on i, as we observe.

Row five illustrates that the small-risk value v_s changes only slightly from \$2.418 to \$2.541 million as η is varied from 3 to 1. The economic value does not change since it does not depend on η. However, the variation in η does have a major effect on p_{max}, decreasing it from 0.140 to 0.0640. For the given lifetime distribution, the larger is η, the less highly the individual values his present life lottery. Hence he will accept smaller payments to incur a given small additional risk and he will be more willing to take very large risks.

Row six demonstrates the effect of age. A 35 year old has a small-risk value of $2.157 million; a 15 year old a small-risk value of $2.671 million. The ratio between v_s and v_e does not change much in this age range. The maximum acceptable probability of death falls from 0.125 for the 35 year old to 0.0920 for the 15 year old. The older you are, the less you have to lose.

A more extensive analysis of the effect of age appears as Table 10.2. Here we see that if the base case individual were 45, his small-risk life value would be about 3/4 of the small-risk value at age 25. At age 60, it is about 1/2 the age 25 value. The ratio of small-risk value to economic value continually decreases, while p_{max} stays about 0.25 beyond age 75. If the base-case individual manages his investments so that his equivalent annual consumption stays at $20,000 throughout his lifetime, then Figure 10.2 shows how the individual's criteria for life-risking decisions change as he moves from childhood through retirement.

As a final sensitivity, we can determine the effect that uncertain lifetime plays in base case results by comparing them with the case where remaining lifetime is certain to be $\bar{\ell} = 46.2$ years. From Equation 8.26 we find $p_{max} = 0.0357$ while from Equation 8.27 we have $v_s = \$3.049$ million. Thus, the uncertainty in lifetime has caused p_{max} to increase by a factor of about three, and has caused the small-risk value v_s to decrease by about 20%.

Table 10.2: The Effect of Age

Age	Expected Remaining Life $\bar{\ell}$ (yr.)	Small-Risk Value v_s(\$million)	Economic Value v_e(\$million)	v_s/v_e	p_{max}
15	55.4	2.671	0.381	7.01	0.0920
25	46.2	2.430	0.363	6.69	0.103
35	36.8	2.157	0.334	6.46	0.125
45	27.8	1.838	0.293	6.28	0.158
55	20.0	1.464	0.242	6.04	0.195
65	13.5	1.080	0.187	5.77	0.228
75	8.4	0.703	0.132	5.31	0.253
85	4.8	0.401	0.084	4.78	0.272
95	4.6	0.227	0.054	4.19	0.266

To assure that the results do not depend heavily on the mortality table used, the computations were repeated for a different mortality table [11] for white males. For the base case individual, expected remaining life is 45.9 years, small-risk value, $2.532 million, and p_{max}, 0.100. These results are within a few percent of our previous values.

The same table contained mortality data for white females. For a female base case individual, expected remaining life is 51.6 years, small-risk value, $2.815 million, and p_{max}, 0.080. The longer expected life of the female causes a substantial increase in small-risk value and a reduction in p_{max}.

11. The White Pill

Our presentation up to this point has emphasized the question of what we must pay an individual to undertake an additional risk. However, the individual and society often face the problem of spending resources to avoid risk or in other words increase safety. The same theoretical model serves to illuminate this problem with only a few small twists.

Suppose that an individual faces a hazard that will kill him with probability p, for example, an operation. If he survives, he will live his normal life with whatever wealth he possesses. However, now someone arrives with a white pill that if taken will surely eliminate the death risk from this hazard. How much, x, would the individual be willing to pay for the white pill? The situation is diagrammed in Figure 11.1.

The unusual feature of this situation is that, of course, x cannot exceed W, since the individual cannot pay more than his wealth for the pill no matter what death risk he faces. We might expect the x versus p curve to look like that of Figure 11.2. The individual will pay more as p increases, but the payment is limited by his wealth.

The simple model corresponding to Figure 8.1 appears as Figure 11.3. Buying the white pill at cost x reduces future consumption to $c - \zeta x$, where ζ is as defined in Section 8. We thus presume that the individual would borrow

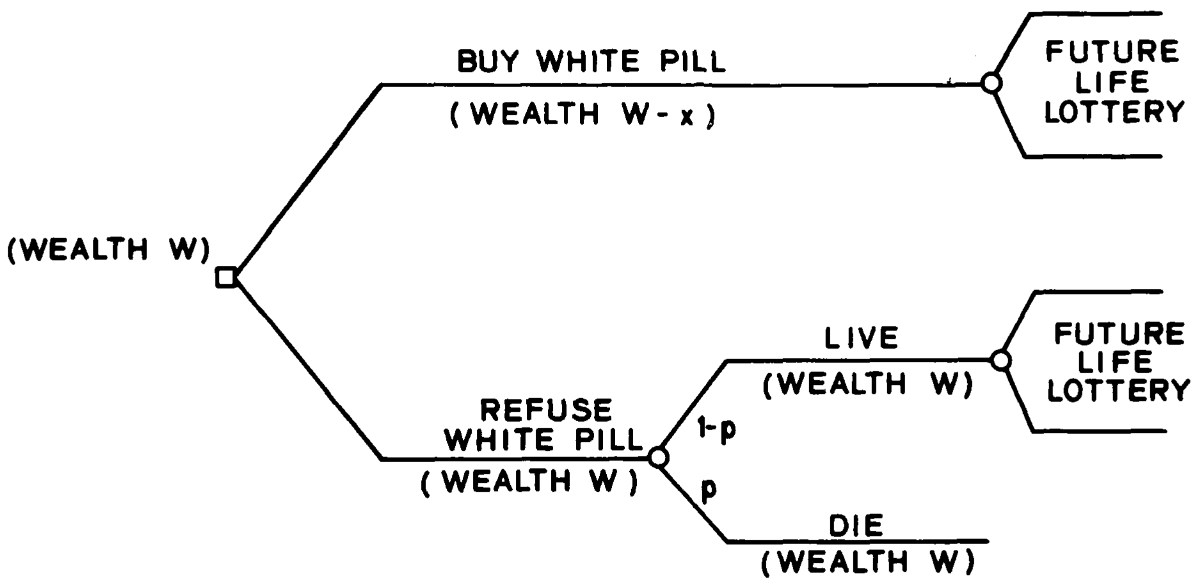


Figure 11.1: The White Pill

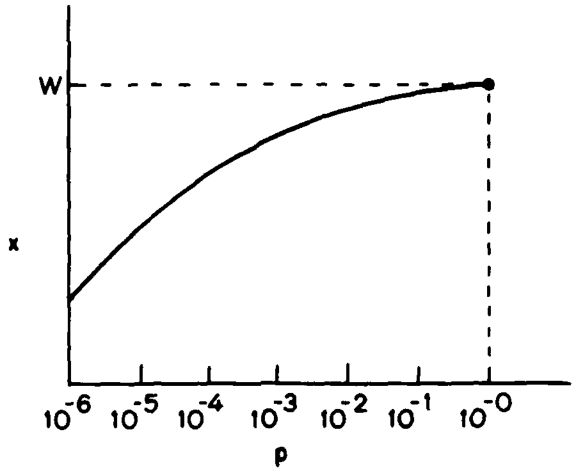


Figure 11.2: The White Pill Tradeoff

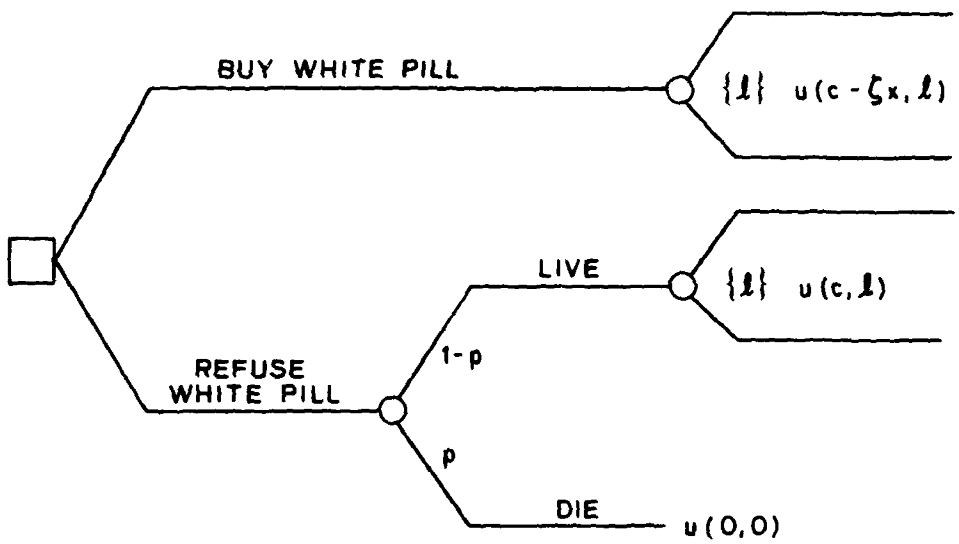


Figure 11.3: A White Pill Model

against future consumption the same amount each year in order to pay x. Of course, ζx must be less than c. If the individual refuses the pill, then he dies with probability p and continues his life of consumption c with probability $1-p$. For him to be indifferent between the two alternatives, we have

$$\left\langle u(c-\zeta x,\ell)\right\rangle = p\, u(0,0) + (1-p)\left\langle u(c,\ell)\right\rangle \tag{11.1}$$

or

$$p = \frac{\left\langle u(c-\zeta x,\ell)\right\rangle - \left\langle u(c,\ell)\right\rangle}{u(0,0) - \left\langle u(c,\ell)\right\rangle} . \tag{11.2}$$

This equation provides the relation between p and x. We observe, as required, that the equation is satisfied by $p = 0$, $x = 0$. The small-risk value of life is again given by

$$v_s = \frac{1}{\left.\frac{dp}{dx}\right|_{\substack{p=0\\x=0}}} , \tag{11.3}$$

and since

$$\frac{dp}{dx} = \frac{-\zeta\left\langle \frac{\partial}{\partial c} u(c-\zeta x,\ell)\right\rangle}{u(0,0) - \left\langle u(c,\ell)\right\rangle} \tag{11.4}$$

we have immediately that

$$v_s = \frac{1}{\left.\frac{dp}{dx}\right|_{\substack{p=0\\x=0}}} = \frac{\left\langle u(c,\ell)\right\rangle - u(0,0)}{\zeta\left\langle \frac{\partial}{\partial c} u(c,\ell)\right\rangle} \tag{11.5}$$

which is the same value of v_s specified by Equation 8.11 for the black pill case. Thus the small-risk value of life is the same whether we consider small increments or decrements in life risk, a satisfying result.

We can again show the reasonableness of this finding by adding the decreased utility of the payment for the white pill when p is small, $- p\, v_s\, \zeta \left\langle \frac{\partial}{\partial c} u(c,\ell) \right\rangle$, to the increased utility from taking it, $\langle u(c,\ell)\rangle - [p\, u(0,0) + (1-p) \langle u(c,\ell)\rangle]$, and setting the sum to zero to reflect the individual's indifference. After dividing by p, we solve for v_s and obtain the result of Equations 8.11 or 11.5.

Now we use the utility function of Equation 8.14 to specify these results for the simple model. From Equation 11.2, we have

$$p = \frac{\left\langle e^{-\gamma(c - \zeta x)\left(\frac{\ell}{\bar{\ell}}\right)^{\eta}} \right\rangle \left\langle e^{-\gamma c \left(\frac{\ell}{\bar{\ell}}\right)^{\eta}} \right\rangle}{1 - \left\langle e^{-\gamma c\left(\frac{\ell}{\bar{\ell}}\right)^{\eta}} \right\rangle} . \tag{11.6}$$

Note that as p approaches one, $c - \zeta x$ must approach zero or

$$x = \frac{c}{\zeta} = v_e \ ,$$

the economic value of life. As death becomes increasingly certain, the individual becomes willing to pay his total economic wealth to avoid it.

We can easily make these calculations for the hypothetical male subject of age 25 and with $c = \$20,000$, $\rho = \$6,000$, $i = 0.05$, and $\eta = 2$. The x versus p curve appears in Figure 11.4. Note that as p approaches one, x approaches $v_e = \$363,000$, as predicted. The same figure shows that the value of life v computed from $x = vp$ is approximately equal to $v_s = \$2,430,000$ for p less than 10^{-3}, but that it approaches v_e as p approaches one. Both the x and the v curves show how the economic situation of the individual limits his tradeoff between consumption and life when p approaches one.

We observe in passing that the sensitivity analysis of Table 10.1 is equally relevant to the white pill case.

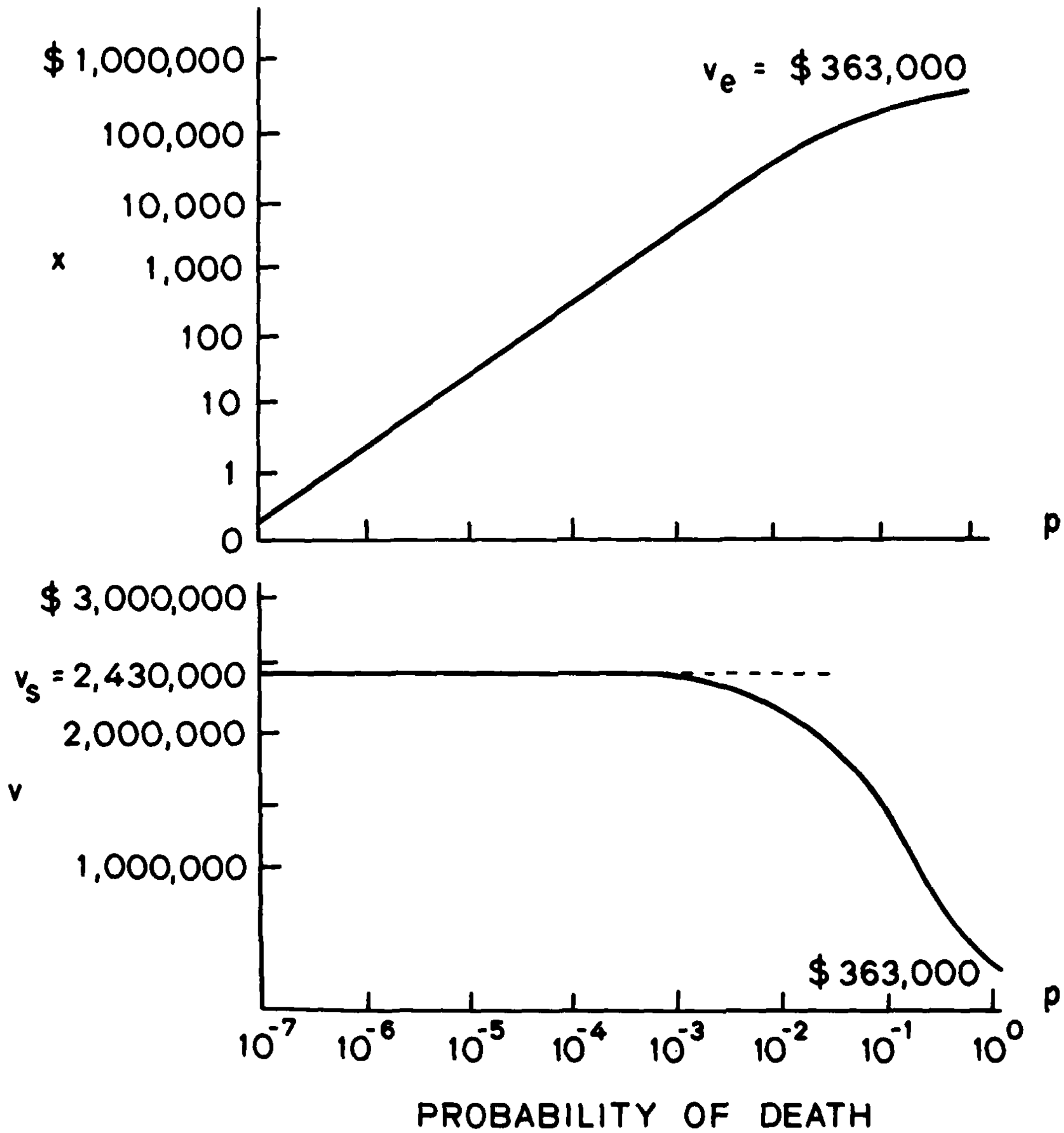


Figure 11.4: White Pill Results

The Value of Risk Reduction

As a consequence of the white pill discussion, we can use the base case individual's small-risk life value to see what he would be willing to pay annually to remove various risks in his life. Table 11.1 shows U.S. accident statistics for 1966 both in terms of number of deaths and death probability. The final column shows what the base case individual would be willing to pay to eliminate these hazards, an amount obtained by multiplying the probability of death by the small-risk life value. Note that he would be willing to pay $900 just to eliminate threats due to motor vehicles and falls. All other sources of accidents contribute collectively to an expected loss of less than $500. This calculation is an important starting point for determining whether feasible safety expenditures to modify these hazards would be worth while. It is clear that spending $1000 to be free of motor vehicle accidents would not be a wise choice for the base case individual. There is a limit to the value of safety.

Table 11.1

U.S. Accident Death Statistics for 1966

Type of Accident	Total Deaths	Probability of Death per Person per Year	Base Case Payment
Motor vehicle	53,041	2.7×10^{-4}	$ 656.00
Falls	20,066	1.0×10^{-4}	243.00
Fire and explosion	8,084	4.0×10^{-5}	97.20
Drowning	5,687	2.8×10^{-5}	68.00
Firearms	2,558	1.3×10^{-5}	31.60
Poisoning (solids and liquids)	2,283	1.1×10^{-5}	26.70
Machinery	2,070	1.0×10^{-5}	24.30
Poisoning (gases and vapors)	1,648	8.2×10^{-6}	19.90
Water transport	1,630	8.1×10^{-6}	19.70
Aircraft	1,510	7.5×10^{-6}	18.30
Inhalation and ingestion of food	1,464	7.3×10^{-6}	17.70
Blow from falling or projected object or missile	1,459	7.3×10^{-6}	17.70
Mechanical suffocation	1,263	6.3×10^{-6}	15.30
Foreign body entering orifice other than mouth	1,131	5.7×10^{-6}	13.90
Accident in therapeutic procedures	1,087	5.5×10^{-6}	13.40
Railway accident (except motor vehicles)	1,027	5.1×10^{-6}	12.40
Electric current	1,026	5.1×10^{-6}	12.40
Other and unspecified	6,163	3.1×10^{-5}	76.50
Total	113,563	5.8×10^{-4}	$ 1,384.00

Russian Roulette

While we are most interested in the implications of our model for the case of small risks, it may be instructive to see what it prescribes in more parlous situations. An interesting example attributed to Richard Zeckhauser concerns Russian Roulette. The game is to place one or more bullets in a six-chambered revolver, spin the cylinder, place the barrel against the temple and pull the trigger. Some game! The question that Zeckhauser poses is whether a player forced to play this game would pay more to remove one bullet from a cylinder containing two bullets than he would to remove one bullet from a cylinder containing only one bullet. Assuming that each bullet represents a 1/6 chance of death, the question is whether the person should pay more to reduce his death probability from 2/6 to 1/6 or from 1/6 to zero.

Table 11.2 shows how much the base case individual would pay to reduce his death probability by 1/6 (remove one bullet) depending on the initial death probability he faced. To remove the last bullet, he would pay about $190,000, which is just the white pill x for $p = 1/6$ from Figure 11.4. To remove the next-to-last bullet, he would pay $208,000 or about $18,000 more.

These figures are consistent, and, of course, do not imply that the individual would pay 208 + 190 = 398 thousand dollars to reduce his risk from 2/6 to zero, for he simply cannot afford that much. To see this we note that when the individual pays 208 thousand dollars to change his death probability from 2/6 to 1/6, his consumption falls

Table 11.2

Payments to Reduce Death Probability by 1/6 in Russian Roulette
(Base Case Individual)

Initial Death Probability	Payment for Reduction by 1/6 (Thousands of Dollars)
1.0	363
5/6	298
4/6	258
3/6	230
2/6	208
1/6	190

from $20,000 per year to $8558 per year. If the individual is now offered a further decrease from 1/6 to 0, he will pay only 51 thousand dollars because of his reduced financial circumstances rather than the 190 thousand dollars he would have paid for this change in his original state. The payment of 51 thousand dollars will leave him with an annual consumption of $5766. The amount that the individual would pay when he has consumption $20,000 to reduce his death probability from 2/6 to 0 is 258 thousand dollars, not 398 thousand dollars. The payment of $258,000 would leave him with an annual consumption of $5766, as required. Incidentally, when our base case individual pays 190 thousand dollars to reduce a 1/6 probability of death to zero, his annual consumption remaining would be $9537. Thus, buying his way out of one-bullet Russian Roulette costs him about one-half his annual consumption.

The table shows that the amount paid to remove a single bullet increases with the number of bullets in the gun. We can explain this by observing that as the number of bullets in the gun increases, the prospects of the individual diminish and hence money is less important to him.

The case of removing one bullet from six is especially interesting because we observe that the individual would pay his whole economic value, $363,000, for this 1/6 decrease in death probability. Of course, if he is facing certain death, and if he is unconcerned about having a legacy, as we assume, he will pay $363,000 for <u>any</u> decrease in death probability since any chance at life is better than none. At this point we would have to examine what we mean by "certain death".

It is alleged that many people find these results counter-intuitive. They think that one should pay more to go from 1/6 to 0 than from 2/6 to 1/6. However, the few people in my life that I have questioned on this point make choices consistent with the implications of the model.

Buying and Selling Hazards

Now that we have both the black pill and white pill results before us, we are in a position to make a few general observations. First, we see that the disparate results of the black and white pill cases for $p = 1$ show that we have answered a continual objection to analyses that place a finite value on life without regard to the black pill/white pill selling hazard/buying hazard distinction. Since few people if any will sell their lives for any finite sum, all such analyses are doomed to failure. However, the present model shows that it is perfectly consistent to refuse any finite offer for your life and yet be limited in what you can spend to save it.

Of greater practical importance, however, is the result that for the wide range of hazardous decisions where we are buying and selling small hazards in our lives, the small-risk life value offers a simple and practical procedure to assure consistency.

12. Continuing Risks

Many of the risks to life occur not at a single instant, as does the black pill, but rather over several years or even a lifetime. The risks of living with automobiles, of smoking, or of living near a power plant are of this type. To analyze them, we must become more specific in characterizing the uncertainty regarding the length of life.

We construct a time scale of regularly-spaced epochs $0,1,2,\ldots$. The period between epochs 0 and 1 we call interval 1, between epochs 2 and 3 interval 2, etc. Typically our interval will be a year.

Let p_n be the probability that an individual will die in interval n given that he is alive at epoch 0. Then p_n for $n = 1,2,3,\ldots$ is the lifetime mass function such as the one shown in Table 9.1 and Figure 9.1. Let q_n be the probability that an individual will die in interval n given that he is alive at epoch $n-1$, where $n = 1,2,3,\ldots$. We call q_n the hazard or, in the case of life, the force of mortality, for q_n shows how likely you are to die in each interval if you were alive at its beginning.

The lifetime mass function and the hazard distribution are simply related by the equations:

$$p_1 = q_1$$

$$p_n = q_n \left[1 - \sum_{j=1}^{n-1} p_j\right] \quad n = 2,3,4,\ldots \tag{12.1}$$

We use these equations to compute the hazard distribution corresponding to a given lifetime mass function or vice versa. Figure 12.1 shows as the solid curve labeled "empirical" the hazard distribution corresponding to the lifetime mass function of Figure 9.1. On this scale the hazard below age 30 is virtually indistinguishable from zero. The points marked "approximation" show the results of the formula

$$q_n = 200 \cdot 2^{\left(\frac{n-30}{9}\right)} .10^{-5} \qquad 30 \leq n \leq 110 \tag{12.2}$$

which closely fit the empirical results. This approximation states that hazard doubles every nine years beyond age 30. It is quite clear why we are not going to escape from this world alive. For purposes of comparison, the expected remaining life of a male after 25 has been computed for both the empirical curve and the approximation; the results differ by less than 2%.

The Value of Hazard Modification

Now we return to our primary interest of investigating continuing risks. Suppose that a person is offered a present sum of money x to incur some modification M of his hazard distribution and hence to face a new lifetime mass function $\{\ell|M\}$. What amount x, positive or negative, would make him indifferent to the proposition? Figure 12.2 shows the choice with the usual assumption that the amount x will be converted into an annuity.

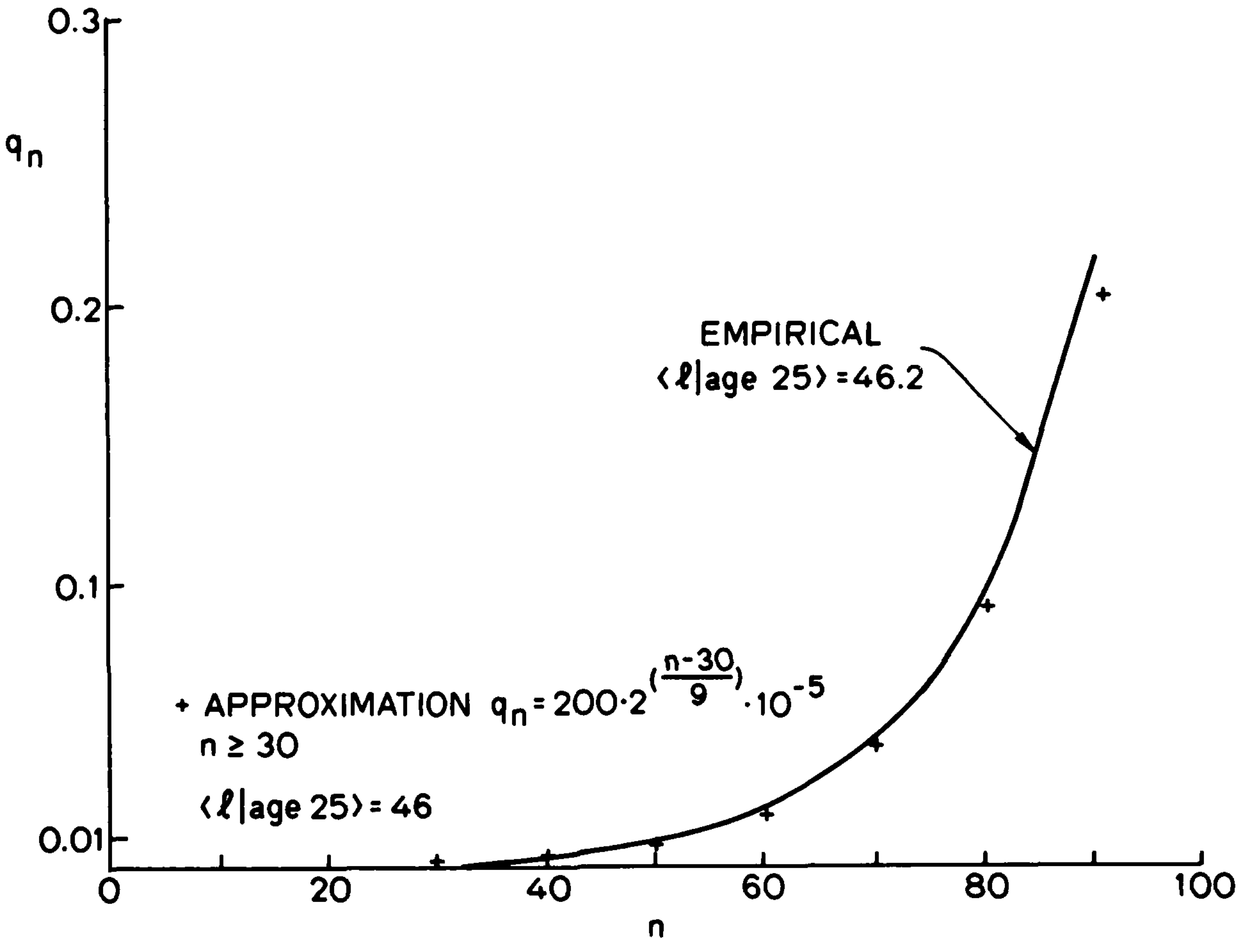


Figure 12.1: The Force of Mortality

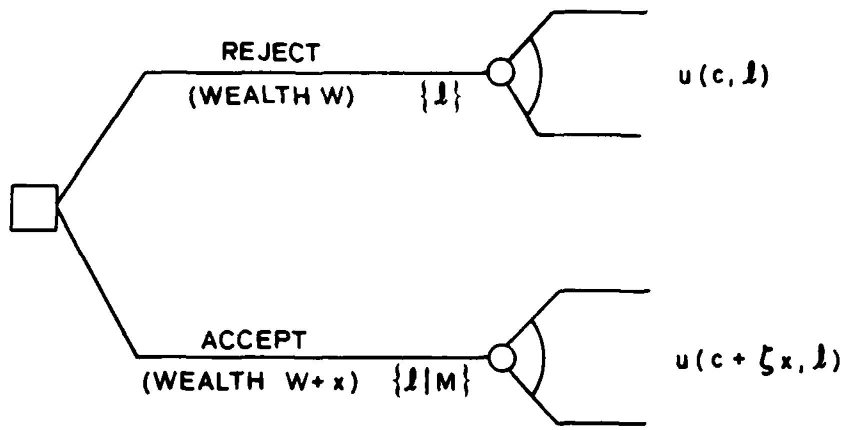


Figure 12.2: Continuing Risk

Let us consider a number of possible hazard modifications. Let M_D be the modification of doubling q_n (with, of course, a requirement that $q_n \leq 1$). Such a serious modification might be comparable to that incurred by smoking heavily throughout life. Let M_H be the modification of halving hazard, a modification that some may believe will result from a regular exercise program to strengthen the cardiovascular system. Let M_A be adding $\frac{1}{4000}$ to every q_n. Such a modification would be comparable to assuming a new risk equal to the present risk from automotive vehicles. Finally, let M_S be subtracting $\frac{1}{4000}$ from every q_n. This would correspond to removing the present risk from automotive vehicles.

We shall consider each of these modifications against the background of our base case (male, age 25, $c = 20{,}000$, $\rho = 6{,}000$ $\eta = 2$, $i = 0.05$). Table 12.1 summarizes the results. Doubling hazard causes the expected lifetime to decrease by 7.8 years and requires a present payment of \$212,000 or a \$12,400 annuity. (We assume that the seller of the annuity is aware of the change in risk when he computes ζ.) Halving the hazard increases life expectancy by 8.0 years and would be worth \$127,000 to the individual, or \$6700 per year. Clearly these are both very significant modifications.

Adding $\frac{1}{4000}$ to hazard decreases life expectancy by 0.3 years and necessitates a payment of \$13,000, or \$700 per year. Subtracting $\frac{1}{4000}$ from hazard increases life expectancy by 0.3 years and would be worth \$12,000 or \$670 per year.

Table 12.1

The Effects of Hazard Modification

Modification: M	$\langle \ell \mid M \rangle$	$\langle \ell \mid M \rangle - \langle \ell \rangle$	x ($)	ζx ($)
M_D: Double q_n	38.4	-7.8	212,000	12,400
M_H: Halve q_n	54.1	8.0	-127,000	-6,700
M_A: Add $\frac{1}{4000}$ to q_n	45.9	-0.3	13,000	700
M_S: Subtract $\frac{1}{4000}$ from q_n	46.4	0.3	-12,000	-670

Thus we see that, as a continuing risk, living in an automotive society costs this individual about \$670 per year in risk cost alone. He should be willing to pay up to this amount for a life "assurance" policy that would assure that he would not be killed in a motor vehicle accident. Of course, all memory of the policy would have to disappear as soon as it was bought to prevent changes in behavior caused by owning the policy.

Note that the \$670 annual risk cost exceeds what the individual would pay to avoid a $\frac{1}{4000}$ black pill risk, since $\frac{1}{4000}\ v_s = \$608$. The continuing nature of the risk makes it more consequential to our base case individual. Nevertheless, the annual compensation for a small change p in annual hazard is conveniently approximated by pv_s.

As a final observation, suppose we consider the modification of changing the hazard pattern by shifting it, and consequently the lifetime mass function, one year into the future, therefore increasing life expectancy by one year. The base case individual would pay \$21,600 as a lump sum for such a change. If we offer him the modification of one year less expected life by shifting in the other direction, he would require a payment of \$23,500 to be indifferent. We thus obtain an indication of what one year of life is worth to this individual.

Delayed Risk

The hazard modification formulation allows us to consider the black pill problem in the case where the payment is made now, but the pill is taken some number of years k in the future. We simply add the black pill death probability p to the normal hazard in year k in the future in the case where both p and the normal hazard are small. Then the paradigm of Figure 12.2 allows us to compute the payment x associated with this modification. If p is small, then we can compute the small risk value $v_s = \lim_{p \to 0} \frac{x}{p}$ and determine how v_s depends upon the delay k. Figure 12.3 shows the result for the base case. The small risk value v_s is \$2.43 million when the delay is zero, about \$2 million when the delay is 10 years, and about \$1 million when the delay is 22 years. By referring to Table 10.2, we find that the v_s value associated with a k year delay of hazard is considerably less than the value of v_s that the individual will have in k years if he survives.

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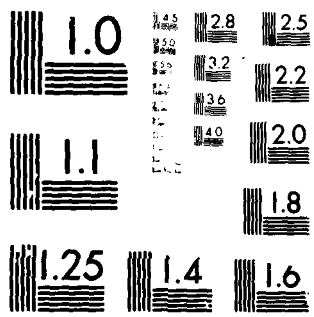

MICROCOPY RESOLUTION TEST CHART
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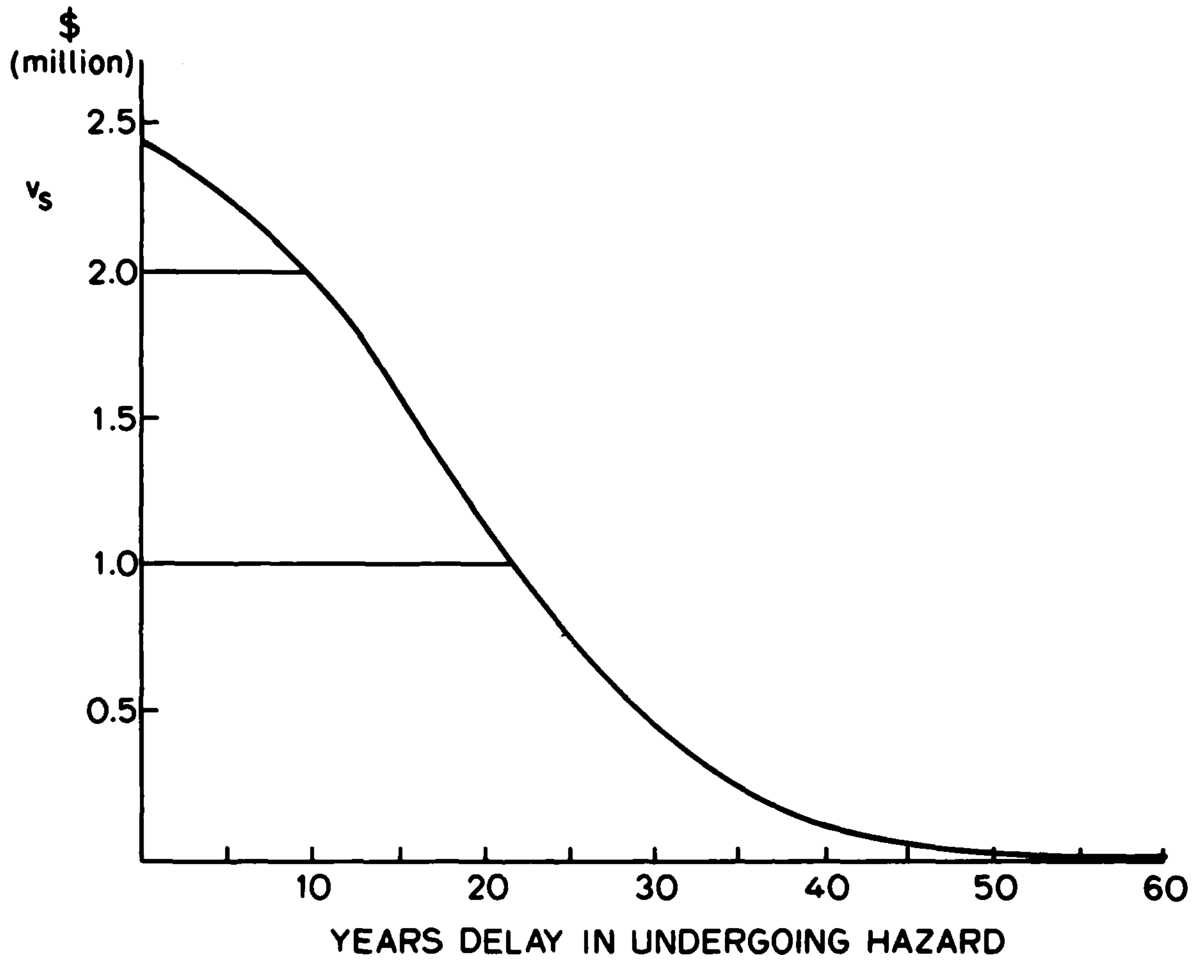

Figure 12.3: Effect of Delay of Risk on v_s

Imposed Modifications of Hazard

We have just discussed the amount that a person would have to be paid to accept a modification of hazard. But suppose that the modification is suddenly imposed - the doctor tells him that he has twice the death risk of a normal person, for example. What will happen to the value he places on his life as revealed by such quantities as v_s and p_{max}? The answer appears in Table 12.2. As the individual faces larger continuing risks in his life, he must decrease v_s and increase p_{max}. He becomes more willing to accept a new risk to his life for a given compensation.

Delegation Revisited

The analysis of continuing hazards shows that specifying a utility function $u(c,\ell)$ on consumption and lifetime not only allows us to solve the black and white pill problems, but to formulate any decision involving length of life and consumption. If the utility function were given to an agent, he could make all decisions within his authority without explicit consideration of a small-risk value of life. Thus, there is no reason to require the agent to make risk-neutral decisions. However, as we have seen, the small-risk-life value is sufficient for the individual or his agent to make most decisions that fall under the rubric of safety.

Table 12.2

The Effect of Hazard Modification on Life Value

Nominal	2.430	0.363	0.103
Modification: M	v_s (millions)	v_e (millions)	p_{max}
M_D: Double q_n	1.433	0.340	0.178
M_H: Halve q_n	4.329	0.380	0.057
M_A: Add $\frac{1}{4000}$ to q_n	2.389	0.362	0.108
M_S: Subtract $\frac{1}{4000}$ from q_n	2.473	0.364	0.098

13. Other Utility Functions

It is interesting to compare our results for the present simple model with those that would be implied by other utility functions in the literature. Barrager [8] suggests a utility function which in our notation would be written as

$$u(c,\ell) = (c\ell)^{\delta} \quad , \tag{13.1}$$

with $\delta = 0.5$. We see immediately from Equation 8.9 that this utility function requires that p_{max} be one. Using Equation 8.11 for v_s yields

$$v_s = \frac{\left\langle u(c,\ell)\right\rangle - u(0,0)}{\zeta \left\langle \frac{\partial}{\partial c} u(c,\ell)\right\rangle} = \frac{\left\langle (c\ell)^{\delta}\right\rangle}{\zeta \left\langle \delta c^{\delta-1} \ell^{\delta}\right\rangle} \quad .$$

If c is not uncertain, we have

$$v_s = \frac{1}{\delta} \frac{c}{\zeta} = \frac{1}{\delta} v_e \quad . \tag{13.2}$$

With $\delta = 0.5$, this utility function implies a small-risk value of life that is twice the economic value.

Usher [9] proposes the utility function

$$u(c,\ell) = c^{\beta} \sum_{j=0}^{\ell} \left(\frac{1}{1+i}\right)^{j} \quad , \tag{13.3}$$

where β is a factor to account for the changing marginal utility of consumption and "is believed to be about 1/3". Again we find that p_{max} equals one. We compute v_s from

$$v_s = \frac{\left\langle c^{\beta} \sum_{j=0}^{\ell} \left(\frac{1}{1+i}\right)^j \right\rangle}{\zeta \left\langle \frac{\partial}{\partial c} c^{\beta} \sum_{j=0}^{\ell} \left(\frac{1}{1+i}\right)^j \right\rangle} \quad .$$

If c is again not uncertain, we have

$$v_s = \frac{1}{\beta} \cdot \frac{c}{\zeta} = \frac{1}{\beta} v_e \quad . \tag{13.4}$$

The small-risk value of life would then be about three times the economic value.

Both of these utility functions therefore yield values of v_s/v_e that are much lower than our base case values. If we refer to Table 10.1, we see that a ratio of 3 would require a ρ/c of about 0.6 and hence from Figure 9.2 a p_d of about 0.6. I have not yet met individuals who have assessed such low values of p_d.

One might be tempted to modify the model by substituting another risk preference curve $u(w)$ for the exponential

$$u(w) = -e^{-\gamma w} = -e^{-w/\rho} \quad .$$

One interesting choice would be the logarithmic utility

$u(w) = \log(\rho + w)$.

But this choice has a problem. As we have said, people typically require a probability of doubling as opposed to halving consumption, p_d, that is well in excess of 0.5. Recall that for our base case individual, p_d is 0.82. With the logarithmic utility curve $u(w) = \log(\rho + w)$, the values of p_d range from 0.333 to 0.5 as ρ decreases from ∞ to 0. Hence no logarithmic utility function can produce p_d's in the desired range.

14. Consumption Uncertainty

While future life lotteries have always been considered in the form $\{c,\ell\}$ that allows a joint uncertainty in both c and ℓ, we have, in fact, not examined the case where the individual is uncertain about his future consumption c. Suppose, for example, that our base case individual faces the lottery on annual consumption shown in Figure 14.1. Instead of $20,000 annual consumption, he assigns equal chances to receiving either $15,000 or $25,000.

We can once more use Equations 8.11 and 8.17 to find v_s and p_{max}; however, now the expectation is taken over the independent uncertainties in c and ℓ. We learn that v_s equals $1.962 million and that p_{max} equals 0.113, compared to $2.430 million and 0.103 for the base case. The small-risk value of life has fallen by almost 20% because of the uncertainty in consumption level, and the individual has increased his maximum acceptable level of risk by 0.01. This is consistent with our earlier observation that v_s will decrease and p_{max} will increase as the future life lottery becomes less desirable to the individual.

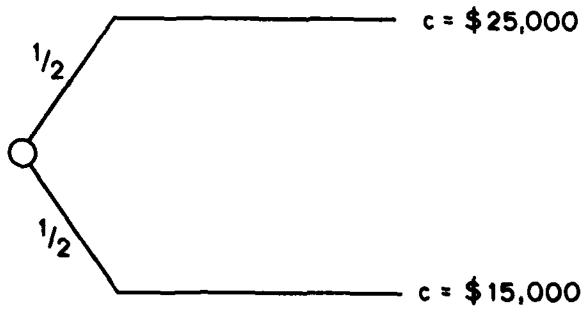

Figure 14.2: Uncertainty in Annual Consumption

15. The Value of the Lives of Others

We may use an extension of this approach to investigate the question of how much one should pay to save the lives of others. Let us suppose that you are asked to contribute for a white pill to be taken by an associate. How much, x, would you pay? We shall use "1" to indicate you and "2" to designate the other person. Figure 15.1 illustrates the situation. If you do not buy the pill 2 will die with probability p. In this case, his remaining life ℓ_2 will be zero, but you will enjoy your present life prospects. If 2 does not die, then you both enjoy your future life lotteries. If you do buy the pill for an amount x, your consumption after buying it will be $c_1 - \zeta_1 x$, your life expectancy will be unchanged and person 2 will be restored to his original consumption-lifetime lottery. We thus require your utility function $u_1(c_1, \ell_1, c_2, \ell_2)$ on your joint prospects, and at the point of indifference we can write

$$\langle u_1(c_1 - \zeta_1 x, \ell_1, c_2, \ell_2)\rangle = p\langle u_1(c_1, \ell_1, c_2, 0)\rangle + (1 - p)\langle u_1(c_1, \ell_1, c_2, \ell_2)\rangle \tag{15.1}$$

or

$$p = \frac{\langle u_1(c_1 - \zeta_1 x, \ell_1, c_2, \ell_2)\rangle - \langle u_1(c_1, \ell_1, c_2, \ell_2)\rangle}{\langle u_1(c_1, \ell_1, c_2, 0)\rangle - \langle u_1(c_1, \ell_1, c_2, \ell_2)\rangle} . \tag{15.2}$$

We can use this equation to solve for x versus p.

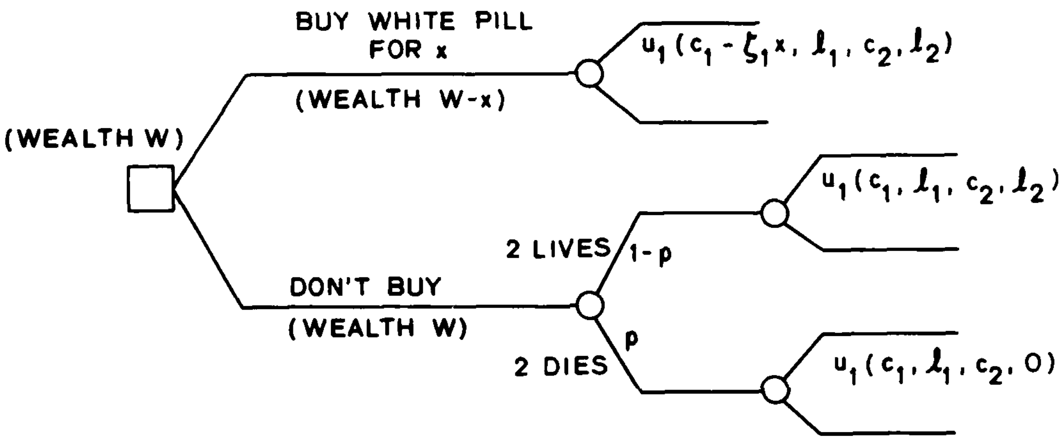

Figure 15.1: The Value of Another

If we are interested in $v_{s_{12}}$, the small-risk value that person 1 assigns to person 2's life, then since for small p, $x = v_{s_{12}} p$, we can obtain $v_{s_{12}}$ from

$$v_{s_{12}} = \frac{1}{\left.\frac{dp}{dx}\right|_{\substack{p=0\\x=0}}} \quad . \tag{15.3}$$

By differentiating Equation 15.2, we find

$$\frac{dp}{dx} = \frac{-\zeta_1 \left\langle \frac{\partial}{\partial c_1} u_1(c_1 - \zeta_1 x, \ell_1, c_2, \ell_2) \right\rangle}{\langle u_1(c_1, \ell_2, c_2, 0)\rangle - \langle u_1(c_1, \ell_1, c_2, \ell_2)\rangle} \tag{15.4}$$

and hence

$$v_{s_{12}} = \frac{\langle u_1(c_1, \ell_2, c_2, \ell_2)\rangle - \langle u_1(c_1, \ell_1, c_2, 0)\rangle}{\zeta_1 \left\langle \frac{\partial}{\partial c_1} u_1(c_1, \ell_1, c_2, \ell_2)\right\rangle} \quad . \tag{15.5}$$

To explore this issue in more detail, we must specify a form for the utility function u_1. Let

$$w_1 = c_1 \left(\frac{\ell_1}{\bar{\ell}_1}\right)^{\eta_1} \qquad w_2 = c_2 \left(\frac{\ell_2}{\bar{\ell}_2}\right)^{\eta_2} \tag{15.6}$$

be the self-worth functions for the two parties, each assessed by our

usual methods. Let

$$w = w_1 + fw_2 \tag{15.7}$$

be the worth function used by party 1 in situations involving outcomes to party 2. We can think of Equation 15.7 as a simple way for party 1 to balance party 2's outcomes against his own; of course, much more complex combinations are possible. The factor f we may think of as a friendship factor. If $f = 1$, party 2's welfare counts as much to 1 as does his own. If $f = 0$, party 2's welfare is a matter of indifference to party 1. We would expect the usual case to be $0 \leq f \leq 1$ with $f > 1$ corresponding to adulation and $f < 0$ to antipathy. We thus have

$$u_1(c_1, \ell_1, c_2, \ell_2) = u(w) = -e^{-\gamma_1 w} = -\exp\left(-\gamma_1\left[c_1\left(\frac{\ell_1}{\bar{\ell}_1}\right)^{\eta_1} + fc_2\left(\frac{\ell_2}{\bar{\ell}_2}\right)^{\eta_2}\right]\right) \tag{15.8}$$

where γ_1 is the risk aversion coefficient on worth for party 1. Since

$$\frac{\partial}{\partial c_1} u_1(c_1, \ell_1, c_2, \ell_2) = \gamma_1\left(\frac{\ell_1}{\bar{\ell}_1}\right)^{\eta_1} \exp\left(-\gamma_1\left[c_1\left(\frac{\ell_1}{\bar{\ell}_1}\right)^{\eta_1} + fc_2\left(\frac{\ell_2}{\bar{\ell}_2}\right)^{\eta_2}\right]\right) , \tag{15.9}$$

we have, from Equation 15.5,

$$v_{s_{12}} \qquad \frac{\left\langle e^{-\gamma_1 c_1 \left(\frac{\ell_1}{\bar{\ell}_1}\right)^{n_1}} \right\rangle - \left\langle e^{-\gamma_1 \left[c_1 \left(\frac{\ell_1}{\bar{\ell}_1}\right)^{n_1} + fc_2 \left(\frac{\ell_2}{\bar{\ell}_2}\right)^{n_2}\right]} \right\rangle}{\zeta_1 \gamma_1 \left\langle \left(\frac{\ell_1}{\bar{\ell}_1}\right)^{n_1} e^{-\gamma_1 \left[c_1 \left(\frac{\ell_1}{\bar{\ell}_1}\right)^{n_1} + fc_2 \left(\frac{\ell_2}{\bar{\ell}_2}\right)^{n_2}\right]} \right\rangle} \quad . \tag{15.10}$$

Note, as required, that $v_{s_{12}} = 0$ when $f = 0$.

If the lifetimes ℓ_1 and ℓ_2 of persons 1 and 2 are independent, then

$$v_{s_{12}} = \frac{\left\langle e^{-\gamma_1 c_1 \left(\frac{\ell_1}{\bar{\ell}_1}\right)^{n_1}} \right\rangle \left[1 - \left\langle e^{-\gamma_1 fc_2 \left(\frac{\ell_2}{\bar{\ell}_2}\right)^{n_2}} \right\rangle\right]}{\zeta_1 \gamma_1 \left\langle \left(\frac{\ell_1}{\bar{\ell}_1}\right)^{n_1} e^{-\gamma_1 c_1 \left(\frac{\ell_1}{\bar{\ell}_1}\right)^{n_1}} \right\rangle \left\langle e^{-\gamma_1 fc_2 \left(\frac{\ell_2}{\bar{\ell}_2}\right)^{n_2}} \right\rangle} \quad . \tag{15.11}$$

Consider the symmetric case where person 1 and person 2 have the same expectations for consumption and life:
$n_1 = n_2 = n$, $c_1 = c_2 = c$, and ℓ_1 and ℓ_2 are independent but with the same distribution $\{\ell\}$.

Then we have

$$v_{s_{12}} = \frac{\left\langle e^{-\gamma_1 c\left(\frac{\ell}{\bar{\ell}}\right)^n} \right\rangle \left[1 - \left\langle e^{-\gamma_1 fc\left(\frac{\ell}{\bar{\ell}}\right)^n} \right\rangle\right]}{\zeta_1\gamma_1 \left\langle \left(\frac{\ell}{\bar{\ell}}\right)^n \; e^{-\gamma_1 c\left(\frac{\ell}{\bar{\ell}}\right)^n} \right\rangle \left\langle e^{-\gamma_1 fc\left(\frac{\ell}{\bar{\ell}}\right)^n} \right\rangle} \; . \tag{15.12}$$

If now $f = 1$, we find $v_{s_{12}} = v_s$ as given by Equation 8.18. Person 1 would value person 2's life for small risks at the same value he assigns to his own.

Returning now to Equation 15.11, we let f approach zero, write

$$v_{s_{12}} \simeq \frac{\left\langle e^{-\gamma_1 c_1\left(\frac{\ell_1}{\bar{\ell}_1}\right)^n} \right\rangle \left[1 - 1 + \gamma_1 fc_2 \left\langle \left(\frac{\ell_2}{\bar{\ell}_2}\right)^{n_2} \right\rangle\right]}{\zeta_1\gamma_1 \left\langle \left(\frac{\ell_1}{\bar{\ell}_1}\right)^{n_1} \; e^{-\gamma_1 c_1\left(\frac{\ell_1}{\bar{\ell}_1}\right)^{n_1}} \right\rangle} \; , \tag{15.13}$$

and find

$$\lim_{f \to 0} \frac{v_{s_{12}}}{f} = \frac{c_2 \left\langle e^{-\gamma_1 c\left(\frac{\ell_1}{\bar{\ell}_1}\right)^{n_1}} \right\rangle \left\langle \left(\frac{\ell_2}{\bar{\ell}_2}\right)^{n_2} \right\rangle}{\zeta_1 \left\langle \left(\frac{\ell_1}{\bar{\ell}_1}\right)^{n_1} \; e^{-\gamma_1 c_1\left(\frac{\ell_1}{\bar{\ell}_1}\right)^{n_1}} \right\rangle} = v_{sd}, \tag{15.14}$$

where v_{sd} is defined as the small-risk, distant-relation value of life. Thus if person 1 has a distant relationship (small f) with a person 2 who faces a small probability of death p, the amount person 1 would contribute to eliminate this risk is approximately pfv_{sd}.

We shall now apply these results to two individuals, each of whom have the base case parameters, but whose lifetimes are independent. Figure 15.2 shows how the life value x/p implied by Equation 15.2 depends on p for varying values of f. For $f = 1$ we obtain the white pill result of Figure 11.4 as expected. For $f = 0.1$ the life value is considerably smaller than 1/10 the life value for $f = 1$ until p approaches 1, when it becomes almost 1/4 of the life value for $f = 1$. For $f = 0.01$, the life value is close to \$10,000 for all values of p.

Figure 15.3 shows how $v_{s_{12}}/f$ computed using Equation 15.11 depends on f. We observe that $v_{s_{12}}/f$ approaches $v_{sd} = \$1,012,500$ as f approaches zero. Thus if our base case individual had a slight relation $(f = 0.01)$ with someone with identical characteristics who faced a 0.01 probability of death, he should contribute about \$100 to eliminate the risk.

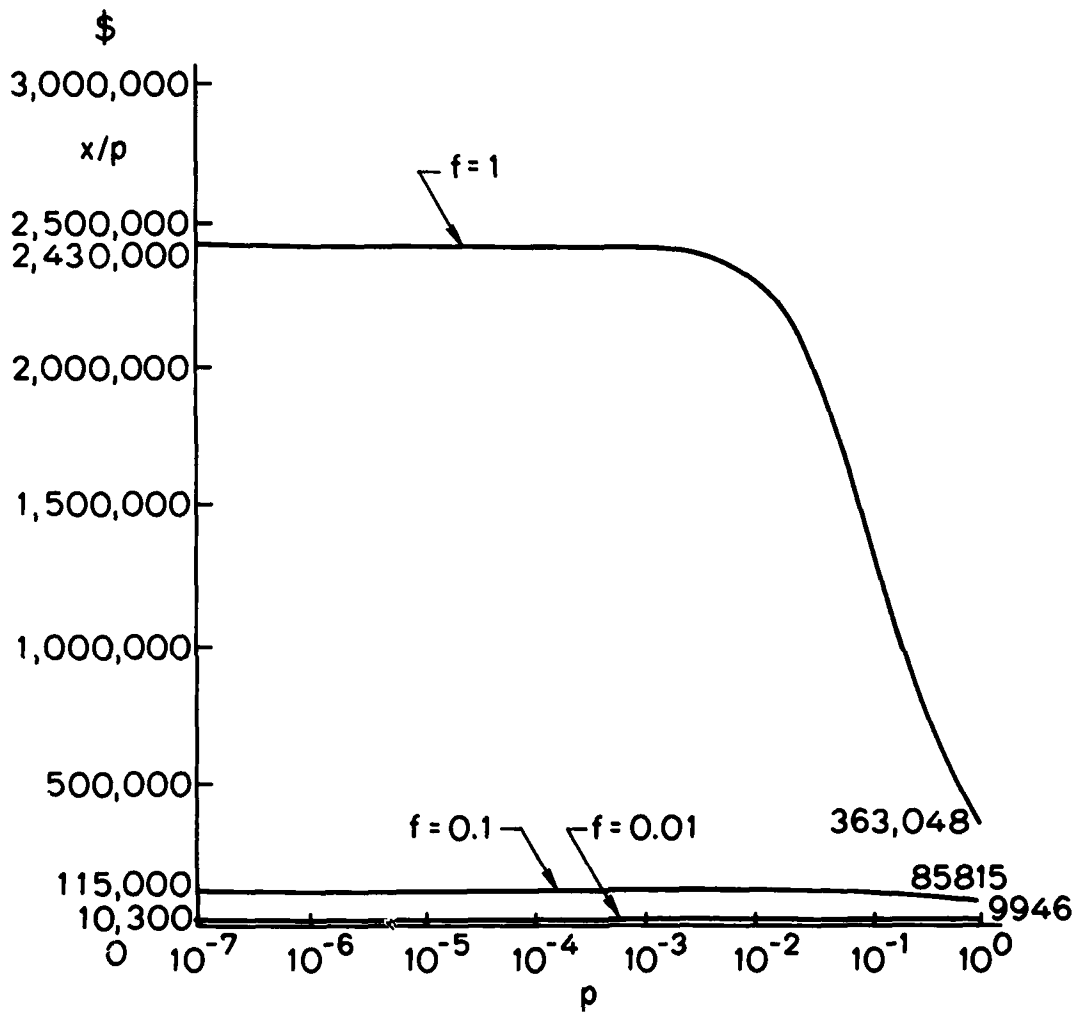

Figure 15.2: Value of the Life of Another

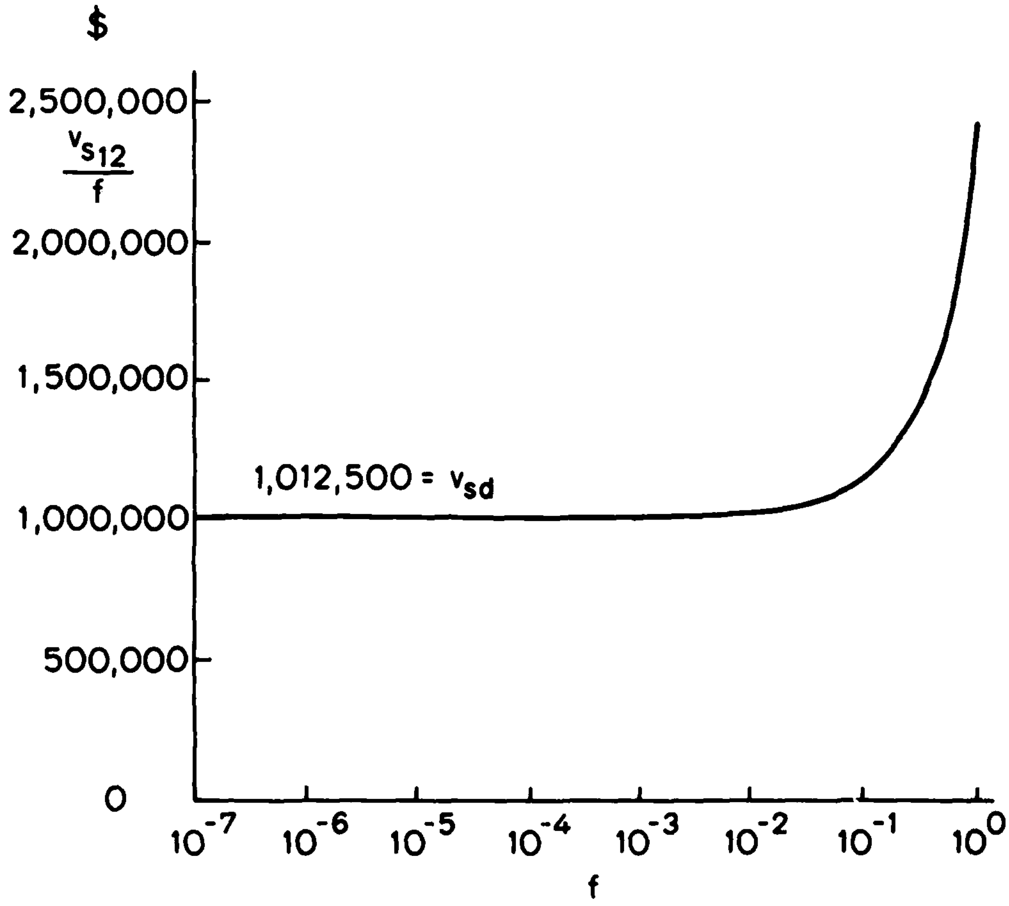

Figure 15.3: Value of the Life of Another - Convergence to the Small Risk-Distant Relation Value

16. Named Lives

One of the perennial questions that arises in this area is whether a "named" life is worth more than a "statistical" life. Society as a whole often seems willing to spend much more to save a particular threatened person than it will to reduce the expected number of deaths by one. We usually encounter these situations where a child has fallen down an abandoned well or someone is lost in the wilderness and is the subject of air search. The existence of effective, but expensive, life-saving procedures like kidney dialysis has caused us to confront this issue much more seriously now than in the past.

To examine the question, let us suppose that you are offered a special type of life-saving insurance. If you find yourself in one of these life-threatening situations, then this insurance will provide financial resources to support an effort to save your life. The threatening situation could be of any nature from being lost in the wilderness to having a medical problem curable at great expense. How much should you pay for the insurance? Let us visualize the situation as shown in Figure 16.1. If you don't buy the insurance, then with probability t you will encounter such a life-threatening situation in the coming year. If you don't encounter the situation, you live your normal life. If you do encounter it, then you live your normal life with probability s_{wo}, the probability that you survive the situation without the insurance. We assume in this case that your survival hinges on other matters than your financial resources, like whether you can find your way out of the wilderness. We could modify the analysis to the case where

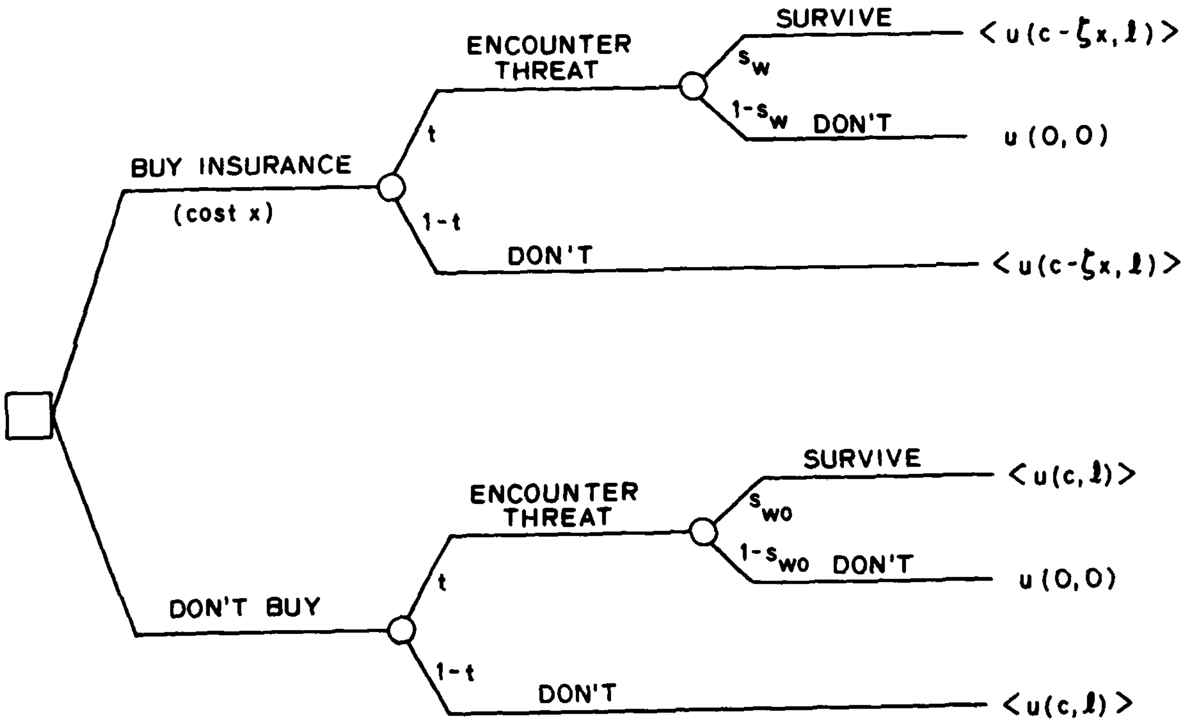

Figure 16.1: Survival Insurance

you can use your financial resources, but we choose not to do so for simplicity.

If you buy the insurance, then you pay the premium x , regardless of what happens; however, now if you encounter the threat there is a probability s_w that you will survive by using the resources provided by the policy. To be indifferent to purchasing the insurance,

$$(1-t + ts_w) \left\langle u(c - \zeta x, \ell) \right\rangle + t(1-s_w) \ u(0, 0) =$$

$$(1-t + ts_{wo}) \left\langle u(c, \ell) \right\rangle + t(1-s_{wo}) \left\langle u(0, 0) \right\rangle \tag{16.1}$$

or

$$\left\langle u(c - \zeta x, \ell) \right\rangle = \frac{t(s_w - s_{wo})}{1-t + ts_w} \ u(0,0) + \frac{1-t + ts_{wo}}{1-t + ts_w} \left\langle u(c, \ell) \right\rangle . \tag{16.2}$$

We see that this equation is the same as Equation 11.1 with $p = \dfrac{t(s_w - s_{wo})}{1-t + ts_w}$. Therefore, when t and hence p is small x must be given by $x = v_s p$ or

$$x = v_s \frac{t(s_w - s_{wo})}{1-t + ts_w} \simeq v_s \ t(s_w - s_{wo}) . \tag{16.3}$$

The premium for the insurance should be approximately the probability of the threat times the small-risk value of life times the increase in survival probability caused by having the insurance. Note that if $s_w = 1$, $s_{wo} = 0$ we have reproduced exactly the white pill results.

Suppose that the insurance provides coverage k. Then if it is to be actuarially feasible,

$$kt < x \approx v_s t(s_w - s_{wo}) \tag{16.4}$$

or

$$k < v_s(s_w - s_{wo}) \; . \tag{16.5}$$

The coverage could never exceed the small-risk life value, and would usually be considerably less.

We can find how each person would determine his level of coverage when $x = kt$ by standard marginal analysis. Let $s(k) = s_w - s_{wo}$ be the increase in survival probability afforded by coverage k. Then the net benefit to the individual from buying k would be

$$v_s ts(k) - tk \tag{16.6}$$

which is maximized when

$$s'(k) = \frac{1}{v_s} \; . \tag{16.7}$$

Coverage is increased until the survival probability increment per dollar falls to $\frac{1}{v_s}$. Of course, if the initial dollar of coverage provides less than $\frac{1}{v_s}$ in survival probability per dollar, and if additional dollars of coverage provide still less, then no insurance

would be bought at all.

As an example, suppose that our base case individual, a flier, is offered a $1,000,000 policy that will provide air searches for him should he be lost in the wilderness. Suppose further that he and the insurance company agree that there is one chance in 10,000 that he will need the policy in any year. Our individual figures that he has 0.1 chance of surviving without the policy and 0.2 chance of surviving with it, should he be lost. The value he computes is then ($2,430,000) (0.1) (0.0001) or $24.30. But the insurance company will not sell the policy for less than $100, so our base case individual would refuse to buy it.

17. Model Implications

Total Risk Preference and Liability Insurance

Individuals sometimes face financial losses that could have devastating effects on their standards of living. For example, if a person is found at fault in a suit claiming damages to the health or life of another person, he could be subject to a liability with long-ranging effects on his financial situation. The structure we have developed for life-threatening situations can often be used to investigate the purchase of liability insurance against larger financial loss.

Suppose, for example, that an individual faces a liability loss λ in a particular year with probability p. How much x should he pay for insurance against this loss? If he buys the insurance, he faces utility $\langle u(c - \zeta x, \ell)\rangle$. If he does not buy, then with probability $1-p$ the loss does not occur and he faces $\langle u(c,\ell)\rangle$. If the loss does occur, we assume that he will use the annuital mechanism to spread it over the rest of his life so that he faces the expected utility $\langle u(c - \zeta\lambda, \ell)\rangle$. To be indifferent between buying the insurance and not buying it, we have

$$\langle u(c - \zeta x, \ell)\rangle = p \langle u(c - \zeta\lambda, \ell)\rangle + (1-p) \langle u(c,\ell)\rangle \quad . \tag{17.1}$$

This equation can be solved exactly to find the premium that could be justified by any loss, including, for example, one that cut consumption in half $(\zeta\lambda = \frac{c}{2})$. Instead of an exact calculation, however, let us observe what happens to this equation if ℓ is

fixed. In this case, we have in our simple model an exponential utility function on the consumptions $c - \zeta x$, $c - \zeta\lambda$, and c. Because of the exponential property, these quantities can as well be $-\zeta x$, $-\zeta\lambda$, and 0. Thus the dollar losses on an annual basis are simply scaled by ζ and then subjected to the utility curve with risk tolerance ρ/ζ or $\rho\nu$. The risk tolerance of this exponential is therefore the risk tolerance on annual consumption multiplied by the discounted expected lifetime ν. For the base case individual $\rho\nu = (6000)(18.15) = \$108,900$. Thus the base case individual is extremely risk tolerant for any lottery that affects only a single year's income.

Compensation for Hazard

Suppose that one individual wishes to impose a risk on another, perhaps by building a potentially dangerous factory in his neighborhood. One way to handle this problem is to have the factory builder compensate those affected according to the principles we have described. Suppose, however, that there is a difference of opinion in the probability assignments. If our base-case individual fears that he is being subjected to a $\frac{1}{1000}$ chance of death per year, he will demand a compensation of about $2400 per year. The factory builder may believe that the risk is only $\frac{1}{100,000}$ and be willing to pay only $24/year. One way to solve this dispute might be to give the factory builder a choice. He could pay the compensation and then be freed from any claims if the factory kills our base-case individual (the case of negligence being excluded). Or he could not pay the compensation and buy insurance that would pay the estate of the base-case individual $2,400,000 should he be killed. Clearly the first choice is preferable to the base-case individual because it will allow him some additional joy to compensate for his additional risk. However, the second choice would be cheaper for the factory builder unless he can convince the affected party that he should accept the builder's probability assessment. The second choice would have the desired effect of inducing responsible behavior on the part of the factory builder and would do much to overcome the question of probability assignment because of the presence of the third party, the insurance company.

We can extend this idea to the case of buying products like automobiles. The manufacturer could state the value of life he used in making his design safety decisions and perhaps even guarantee to pay that value to the estate of anyone killed as a result of the design. This would be a way to make careful decisions about product safety a positive selling point for the product.

Tort

While there are obvious advantages in reaching agreement about compensation in the case of death before the fact of death, there will inevitably arise situations where one person has been charged to be responsible for another's accidental death and a court is now being asked to fix an amount of damages. Since the deceased is most likely to have refused any finite amount of money for the right to kill him, this problem has always been a major difficulty for the courts. We have already mentioned how the economic value of life has been used in such situations even though it rests on the individual's value to others rather than to himself. It would seem far wiser to proceed along the lines we have indicated.

Suppose, for example, that Mr. Smith had inherited a painting done by his mother. It has a simple, attractive style, but is viewed by others as definitely the work of an ungifted artist. However, Mr. Smith values the picture very highly: he installs burglar alarms, provides special humidity and temperature control systems to protect it from environmental changes, and equips the room containing it with a fire suppression system that will not harm the picture.

One day a business associate, Mr. Jones, visits Mr. Smith at his home; they engage in a heated argument. At one point, Mr. Smith leaves the room in frustration and Mr. Jones, in his anger, falls upon the picture and destroys it. Now the matter has come to court; Mr. Smith seeks damages from Mr. Jones. While admitting that Mr. Jones

is at fault, his lawyers introduce evidence that damages should be very low. Art experts testify that the picture is worth less than \$100 commercially. The point is made that Mr. Smith carried no insurance on the picture. Mr. Smith argues that he did not carry insurance because no sum of money could have replaced the picture. Other art experts testify that the level of care provided by Mr. Smith is usually reserved for pictures worth \$1,000,000. The question now is whether restitution to Mr. Smith is more nearly afforded by an award near \$100 or by an award near \$1,000,000. I would argue that the second figure is more appropriate.

Thus, in the case of death, if there is evidence that a person has always acted as if his life were worth v_s = \$2.4 million when facing small risks, then that is the appropriate sum that should be awarded as a minimum in the event of his death from what he considered to be a low-probability risk. It is interesting to note that juries often award several times economic value and that these awards are regarded by some as excessive. I suggest that the juries' intuition may have been in advance of our analytic methods.

18. Model Refinement

The simple model we have discussed appears useful when the individual is facing small probabilities of death against a background of his normal life situation, for the preferences are used in a setting close to the one in which they are assessed. However, the preference model also could be applied to a far different situation, for example one where the individual faces not a normal life expectancy, but equal probabilities of living one or two more years. We can see the need to assure that the model would be appropriate in such a situation. We foresee that in some cases a new preference assessment would be necessary, perhaps at a much greater level of detail*, year by year and income level by income level, and including other attributes like state of health. We shall not go to this extreme but shall examine some simple additions to the basic model that may extend its range of usefulness without excessive complication. We shall attempt to make the preference model an adequate descriptor of preferences not only in the "normal" situation, but also in the unhappy circumstance where either life is short or income is low.

Consumption-Life Tradeoffs

We shall consider first the tradeoff between consumption and lifetime currently described by the worth function

$$w(c,\ell) = c\left(\frac{\ell}{\bar{\ell}}\right)^{\eta} . \tag{18.1}$$

With $\eta = 2$ this function prescribes that the individual will be indifferent to a 1% decrease in lifetime if he is given a 2% increase in consumption, which may be reasonable. However, the model also implies

*Detailed preference investigations are described in [8] and [10].

that if he currently has two years to live at some consumption level, he will be indifferent to the idea of living only one year if he receives four times the level of consumption. That may be true, but the conclusion should be checked with the individual. If he does not agree with this implication, then we need a richer model to capture his preferences.

To develop such a model, let us define

$$\varepsilon(\ell) = -\frac{dc/c}{d\ell/\ell} \tag{18.2}$$

as the elasticity of consumption with respect to lifetime. For the model of Equation 18.1, we find $\varepsilon(\ell) = \eta$; the elasticity of consumption with respect to lifetime is a constant. Individuals may want this elasticity to change over life in some way. If we allow some arbitrary $\varepsilon(\ell)$ we have

$$-\frac{dc}{c} = \varepsilon(\ell)\frac{d\ell}{\ell}$$

$$c(\ell) = ke^{-\int\frac{\varepsilon(\ell)}{\ell}d\ell}$$

$$= kh(\ell), \text{ where } k \text{ is a constant.} \tag{18.3}$$

Suppose that elasticity is a linear function of ℓ

$$\varepsilon(\ell) = \eta_0 + (\eta_1 - \eta_0)\frac{\ell}{\bar{\ell}} \quad . \tag{18.4}$$

This relation implies that the elasticity is η_0 at $\ell = 0$ and η_1 at $\ell = \bar{\ell}$. Then

$$c(\ell) = k e^{-\eta_0 \ln \ell - (\eta_1 - \eta_0)\frac{\ell}{\bar{\ell}}}$$

$$= k\,\ell^{-\eta_0}\, e^{-(\eta_1 - \eta_0)\frac{\ell}{\bar{\ell}}}$$

$$= kh(\ell) \tag{18.5}$$

where

$$h(\ell) = \ell^{-\eta_0}\, e^{-(\eta_1 - \eta_0)\frac{\ell}{\bar{\ell}}} \ . \tag{18.6}$$

To place this result in the form of a worth function, we note that when $c(\ell) = kh(\ell)$,

$$w(c,\ell) = w(kh(\ell),\ell) = w(kh(\bar{\ell}),\bar{\ell}) = kh(\bar{\ell}) \ ,$$

since we have taken our numeraire to be the consumption at expected lifetime. From the first of these equations $c = kh(\ell)$ and hence

$$w(c,\ell) = c\,\frac{h(\bar{\ell})}{h(\ell)}$$

$$= c\left(\frac{\ell}{\bar{\ell}}\right)^{\eta_0} e^{-(\eta_1 - \eta_0)\left(1 - \frac{\ell}{\bar{\ell}}\right)} \ . \tag{18.8}$$

Of course, this result reduces to our earlier worth function when $\eta_0 = \eta_1 = \eta$. But now we can fit a wider range of preference. Our base case individual, for example, finds that he likes $\eta_0 = 1$, $\eta_1 = 2$. This choice matches his earlier one when ℓ is near $\bar{\ell}$,

but also allows him to express his belief that he would give up only about one-half his consumption to extend his lifetime from one year to two years $(w(\frac{c}{2},2) = 1.02\ w(c,1))$.

Figure 18.1 shows this consumption-lifetime tradeoff function. We observe, for example, that $20,000 per year consumption with 40 years remaining is valued as highly as $61,000 per year consumption with 20 years remaining. It would have required $80,000 per year for 20 years to be indifferent with the original tradeoff function of Figure 9.3. If still more flexibility is needed, Equation 18.3 can be used to generate many other worth functions.

Risk Attitude

We have treated risk attitude very simply by examining a constant risk tolerance ρ. We could, however, let the risk tolerance depend on the worth level w in a linear fashion,

$$\rho(w) = \rho + \phi w \quad , \qquad \rho > 0 \ . \tag{18.9}$$

The risk aversion coefficient $r(w) = 1/\rho(w)$ will increase or stay constant in worth as long as $\phi \geq 0$, which is the case we shall assume. Since

$$r(w) = -\frac{u''(w)}{u'(w)} \quad . \tag{18.10}$$

We can develop the well-known relation for deriving the utility function from the risk tolerance

$$\ln u'(w) = -\int r(w)$$

$$u(w) = \int e^{-\int r(w)} \ .$$

The value $\phi = 0$ produces the exponential utility function

$$u(w) = -e^{-w/\rho} \ ; \tag{18.11}$$

the value $\phi = 1$, the logarithmic utility function

$$u(w) = \ell n(\rho + w).$$

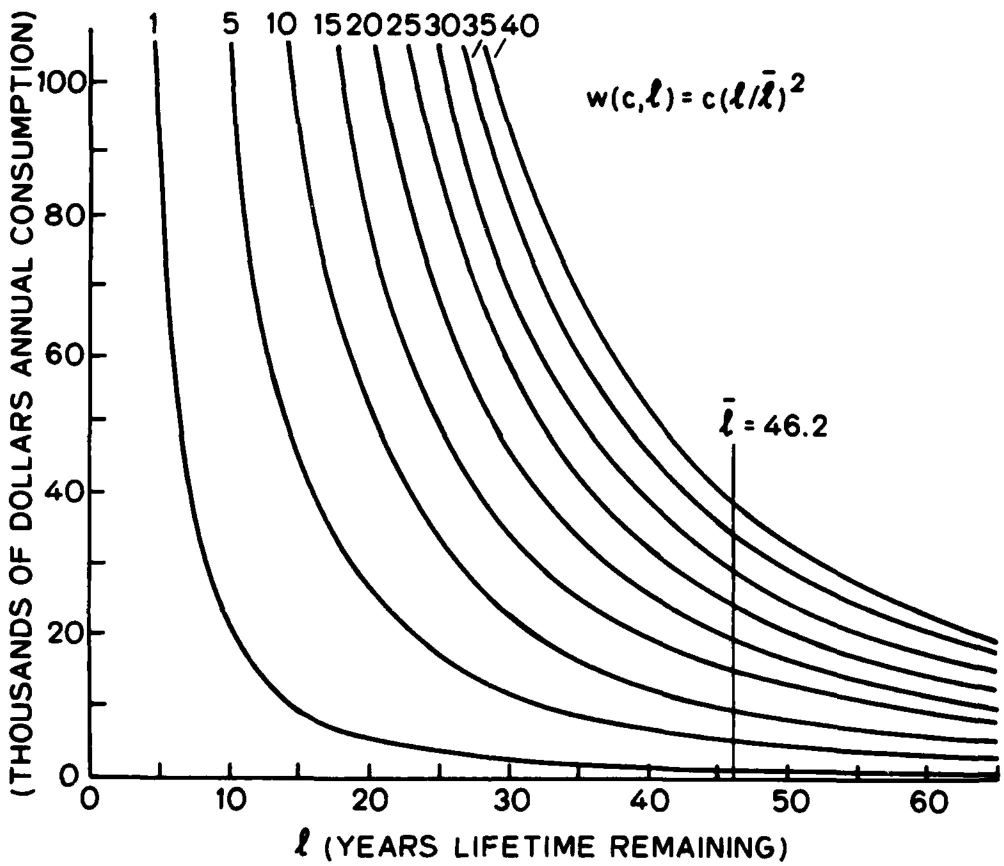

Figure 18.1: A Consumption-Lifetime Tradeoff Function

For other possible ϕ's we have

$$u(w) = \operatorname{sgn}(\phi - 1)(\rho + \phi w)^{1 - \frac{1}{\phi}} . \tag{18.12}$$

For the case of most importance to us, $0 < \phi < 1$,

$$u(w) = -(\rho + \phi w)^{1 - \frac{1}{\phi}} . \tag{18.13}$$

The question now is how to assess ρ and ϕ. It is easy to see for this class of utility functions, that to a good approximation, if the individual is indifferent between a state of worth w and an equiprobable lottery on $w + z$ and $w - z/2$, then $z \simeq \rho(w)$, the risk tolerance at worth level w. Thus, if we assess $\rho(w)$ at two different worth levels a and b we have

$$\rho(a) = \rho + \phi a, \quad \rho(b) = \rho + \phi b. \tag{18.14}$$

We can find ϕ from

$$\phi = \frac{\rho(b) - \rho(a)}{b - a} \tag{18.15}$$

and ρ by substituting this result in either of the original equations.

Our base case individual has assessed $\rho(20{,}000) = 6000$. He also says that if he had worth 5000, he would be indifferent between that situation and a 50-50 lottery on 8000 and 3500. Thus $\rho(5000) = 3000$. When we solve for the ρ and ϕ implied by these results, we find

$$\rho = 2000 \qquad \phi = 0.2 . \tag{18.16}$$

Of course, we can use this scheme only to encode assessments for which $\rho > 0$.

If the linear functional form of Equation 18.9 does not provide enough freedom, one can always resort to a direct encoding of the utility curve.

Attribute Risk Aversion

To gain insight into the preference model, it is interesting to examine the risk aversion on each attribute such as c or ℓ rather than simply the risk aversion on worth. Let r_z^f be the risk aversion coefficient for the attribute z derived from the preference function f,

$$r_z^f = \frac{-\dfrac{\partial^2 f}{\partial z^2}}{\dfrac{\partial f}{\partial z}} \quad . \tag{18.17}$$

Since we have an explicit expression for $u(c,\ell)$, we could derive r_c^u and r_ℓ^u directly from this equation. It is more insightful, however, to think separately of $u(w)$ and $w(c,\ell)$ and to determine how much of the attribute's risk aversion has its source in the risk preference function on the numeraire and how much in the non-linearity of $w(c,\ell)$. Thus to compute r_z^u we write

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial w} \cdot \frac{\partial w}{\partial z}$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial w}\right)\frac{\partial w}{\partial z} + \frac{\partial u}{\partial w}\,\frac{\partial^2 w}{\partial z}$$

$$= \frac{\partial^2 u}{\partial w^2} \cdot \left(\frac{\partial w}{\partial z}\right)^2 + \frac{\partial u}{\partial w}\,\frac{\partial^2 w}{\partial z^2} \tag{18.18}$$

and then

$$r_z^u = \frac{-\frac{\partial^2 u}{\partial z^2}}{\frac{\partial u}{\partial z}} = \frac{-\frac{\partial^2 u}{\partial w^2}}{\frac{\partial u}{\partial w}} \cdot \frac{\partial w}{\partial z} - \frac{\frac{\partial^2 w}{\partial z^2}}{\frac{\partial w}{\partial z}}$$

$$= r_w^u \cdot \frac{\partial w}{\partial z} + r_z^w \quad . \tag{18.19}$$

Thus the risk aversion coefficient for an attribute is the risk aversion coefficient on worth multiplied by the scale change between the attribute and worth plus the risk aversion coefficient of the worth function on that attribute. We can think of this latter term as the induced risk aversion for the attribute.

Let us begin with consumption and compute

$$r_c^u = r_w^u \frac{\partial w}{\partial c} + r_c^w . \tag{18.20}$$

For the linear risk tolerance of Equation 18.9,

$$r_w^u = \frac{1}{\rho + \phi w} . \tag{18.21}$$

From Equation 18.8,

$$\frac{\partial w}{\partial c} = \left(\frac{\bar{\ell}}{\ell}\right)^{n_0} e^{-(n_1 - n_0)}\left(1 - \frac{\ell}{\bar{\ell}}\right) \tag{18.22}$$

and

$$r_c^w = 0 \tag{18.23}$$

since $\frac{\partial^2 w}{\partial c^2} = 0$. Because $\frac{\partial w}{\partial c}$ is a constant in c the risk aversion coefficient on c is just r_w^u times this positive constant. Therefore if the individual has a risk averse utility function u, he will be risk averse on lotteries that affect only the value of c.

Turning now to lifetime, we write

$$r_\ell^u = r_w^u \frac{\partial w}{\partial \ell} + r_\ell^w . \tag{18.24}$$

We need to compute $\frac{\partial w}{\partial \ell}$ from Equation 18.8,

$$\frac{\partial w}{\partial \ell} = \frac{c}{\bar{\ell}} \left(\frac{\ell}{\bar{\ell}}\right)^{n_0-1} e^{-(n_1-n_0)\left(1-\frac{\ell}{\bar{\ell}}\right)} \left[n_0 + (n_1-n_0)\frac{\ell}{\bar{\ell}}\right] . \tag{18.25}$$

This quantity changes with ℓ but is always positive. However,

$$r_\ell^w = -\frac{1}{\ell}\left[n_0 + (n_1 - n_0)\frac{\ell}{\bar{\ell}} - \frac{n_0}{n_0 + (n_1-n_0)\frac{\ell}{\bar{\ell}}}\right] \tag{18.26}$$

may sometimes be negative and it is possible that it be so negative that r_ℓ^u will become negative in some range. For example, in our original model $n_1 = n_0 = n$, $\phi = 0$, and we have

$$r_\ell^u = \frac{1}{\rho} \frac{nc}{\bar{\ell}}\left(\frac{\ell}{\bar{\ell}}\right)^{n-1} - \frac{n-1}{\bar{\ell}} \tag{18.27}$$

which will be negative if

$$\frac{\ell}{\bar{\ell}} < \left[\frac{\rho}{c}\left(\frac{\eta-1}{\eta}\right)\right]^{\frac{1}{\eta}} \tag{18.28}$$

or for the base case, $\ell/\bar{\ell} < 0.387$, or $\ell <$ 17.8 years. Thus the individual would be risk preferring for lotteries on life alone that had outcomes in this range.

Risk preference on an attribute is not necessarily a problem. For example, people often say that at fixed consumption levels they would prefer their present lottery on length of life to a certainty of living the expected lifetime. They would have to be offered a life longer than the expected lifetime for certain before they will accept the lottery; this is risk preference.

The question is really one of measuring the preferences of the individual. If we have a rich enough model, most people's preferences can be encoded. Examining attribute risk preference may be helpful in seeing the implications of the choices.

Numerical Results

We clarify the implications of this section by examining numerous results. Table 18.1 shows the small risk life value and p_{max} for various choices of value function parameters η_0 and η_1 and for various risk attitudes. The first column corresponds to the exponential utility function with $\rho = 6000$. The entries in this column for $\eta_0 = \eta_1 = 2$ correspond to the original base case results. We observe that changing η_0 to 1, to reflect the newly experienced tastes of the base case individual, increases the small-risk value slightly to 2.456 million dollars and decreases p_{max} to 0.0916. In general, for the exponential case we see that changes in η_0 and η_1 between the values 1 and 2 have much greater effects on p_{max} than they do on v_s.

The other columns of the Table correspond to two linear risk tolerance utility functions. One has the parameters $\rho = 2000$, $\phi = 0.2$ derived in Equation 18.16. The other has parameters $\rho = 4000$, $\phi = 0.1$ and represents an intermediate case between this one and the exponential. With $\rho = 4000$, $\phi = 0.1$ we have $\rho(20{,}000) = 6000$ as before, but $\rho(5000) = 4500$. This means that an individual with worth 5000 would be just indifferent between this situation and a 50-50 chance at 2750 and 9500. As we move to the right in Table 18.1, risk aversion for small worths (less than \$20,000) is increasing, while risk aversion for large worths is decreasing.

Since the small-risk life value and p_{max} are both heavily influenced by the degree of risk aversion for small worths, it is not surprising to see

Table 18.1: Effect of Value Function and Risk Attitude on Small-Risk Value and Threshold Death Probability

			Exponential $\rho = 6000$		Linear $\rho = 4000$ $\phi = 0.1$		Linear $\rho = 2000$ $\phi = 0.2$	
	n_0	n_1	v_s*	p_{max}	v_s	p_{max}	v_s	p_{max}
	1	1	2.541	0.0640	3.622	0.0485	7.519	0.0260
	1	2	2.456	0.0916	3.257	0.0720	5.929	0.0422
Base Case:	2	2	2.430	0.1031	3.193	0.0832	5.661	0.0524
	2	1	2.423	0.0774	3.411	0.0607	6.824	0.0358

* v_s is in millions of dollars

v_s increase and p_{max} decrease as we move to the right in Table 18.1. For the case $n_0 = 1$, $n_1 = 2$, $\rho = 2000$, $\phi = 0.2$ we find $v_s = 5.929$ million dollars, almost 2-1/2 times the base case values. To the extent that individuals are more risk averse for small worths than the base case parameters would indicate, the small-risk life values may be understated by factors of 2 or 3.

Selecting a value function for a utility function requires matching the preference of the individual both in general form and in specific number. To do this it is helpful to examine the general characteristics of the function choices that appear in the Table. Figure 18.2 shows the risk aversion on lifetime that is implied by each choice of value function and utility function parameters. Figure 18.2 (a) shows the results for the value parameters $n_0 = 1$, $n_1 = 1$. We see constant or decreasing risk aversion for all utility functions, and no region of risk preference on lifetime. Figure 18.2 (b) for $n_0 = 1$, $n_1 = 2$ shows increasing risk aversion, and, in the exponential case, a region of risk preference. Figure 18.2 (c) for $n_0 = n_1 = 2$ shows generally risk increasing behavior except for the case $\rho = 2000$, $\phi = 0.2$ where risk aversion first increases and then decreases. All of these curves show an initial range of risk preference. As we discussed above, this region extends to 17.8 years for the exponential case. Figure 18.2 (d) shows the highly variable behavior associated with the choice $n_0 = 2$, $n_1 = 1$. Again we observe a region of risk preference.

Within the possibilities examined, a person wishing to have no region of risk preference would be constrained to only 5 of the 12 possibilities examined. For example, the choice $\rho = 4000$, $\phi = 0.1$

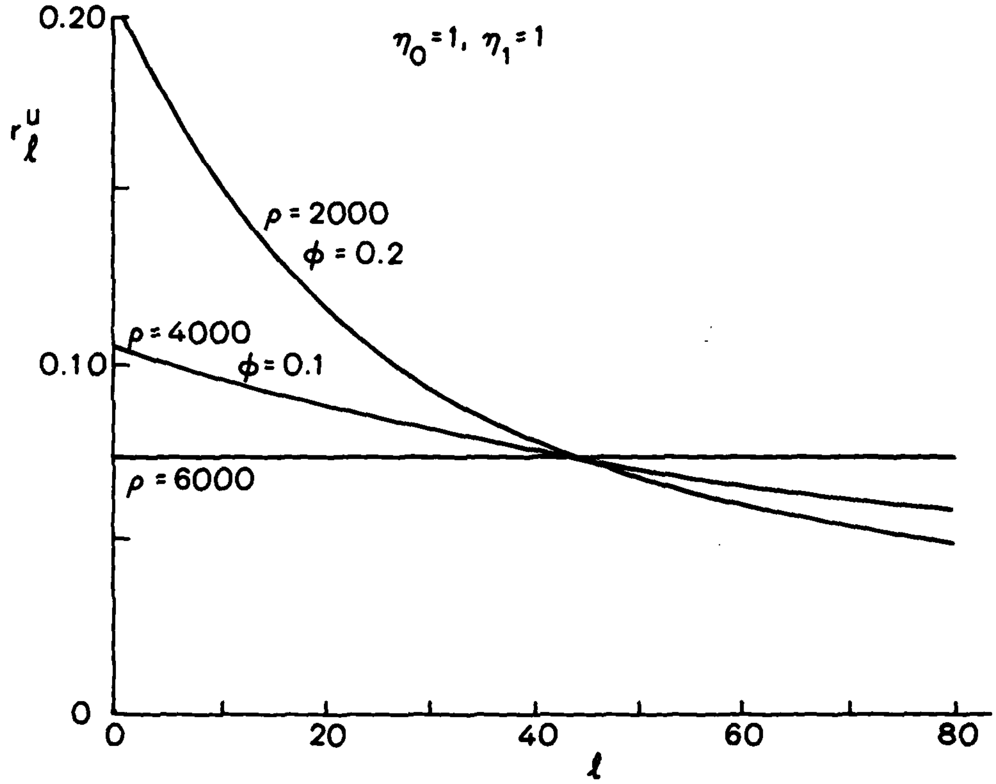

Figure 18.2 (a): Risk Aversion on Lifetime

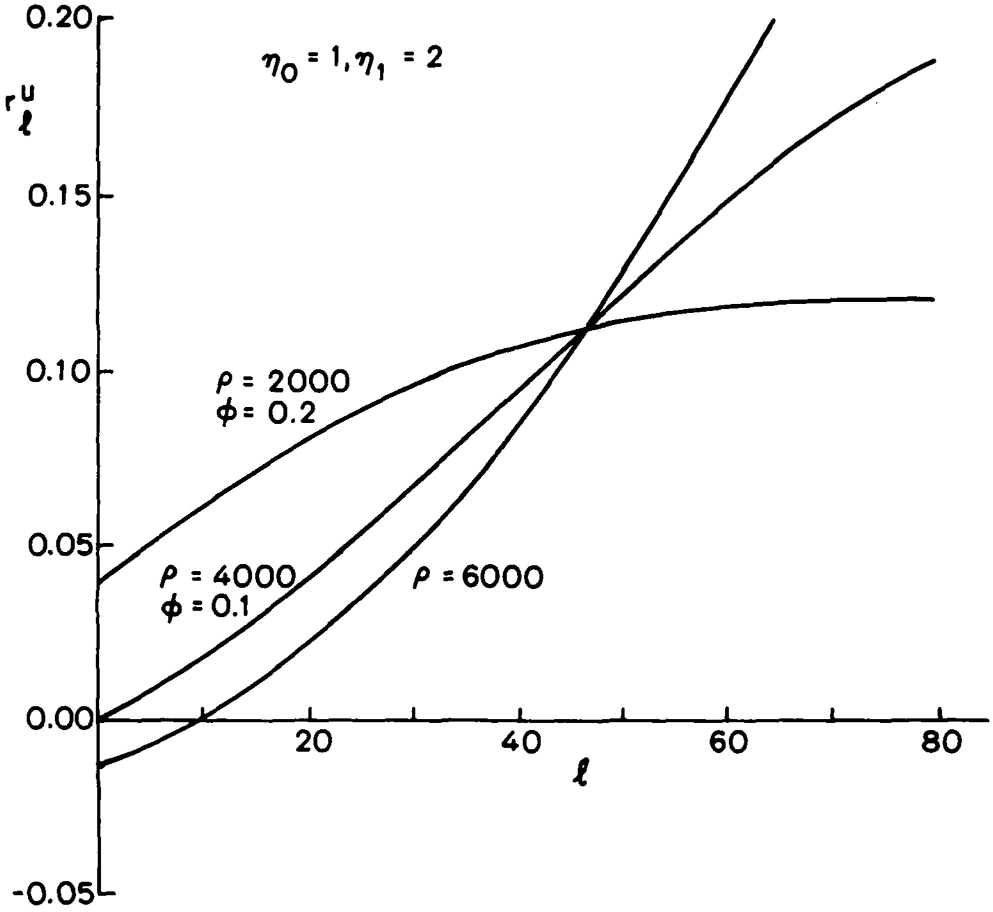

Figure 18.2 (b): Risk Aversion on Lifetime

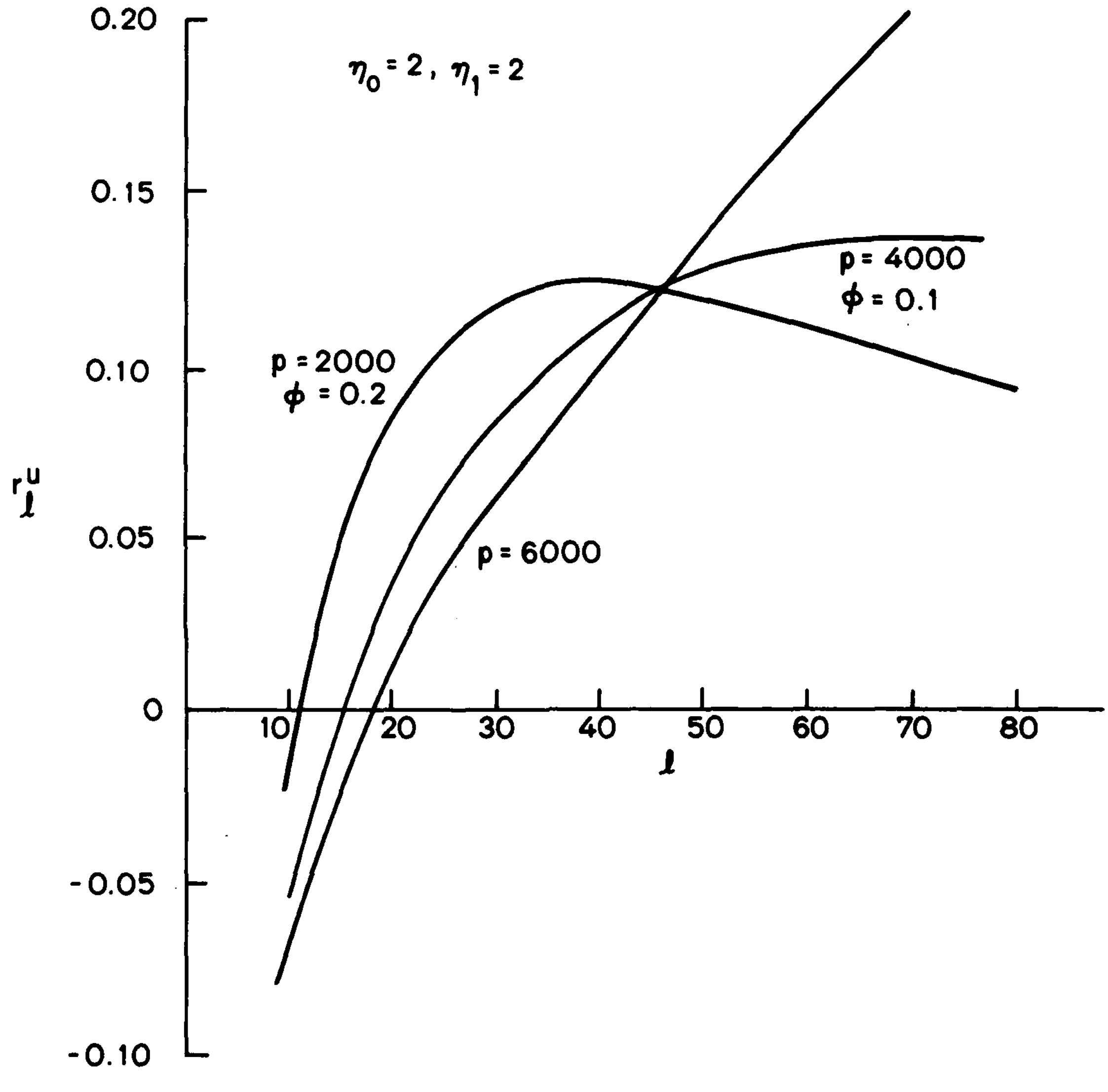

Figure 18.2 (c): Risk Aversion on Lifetime

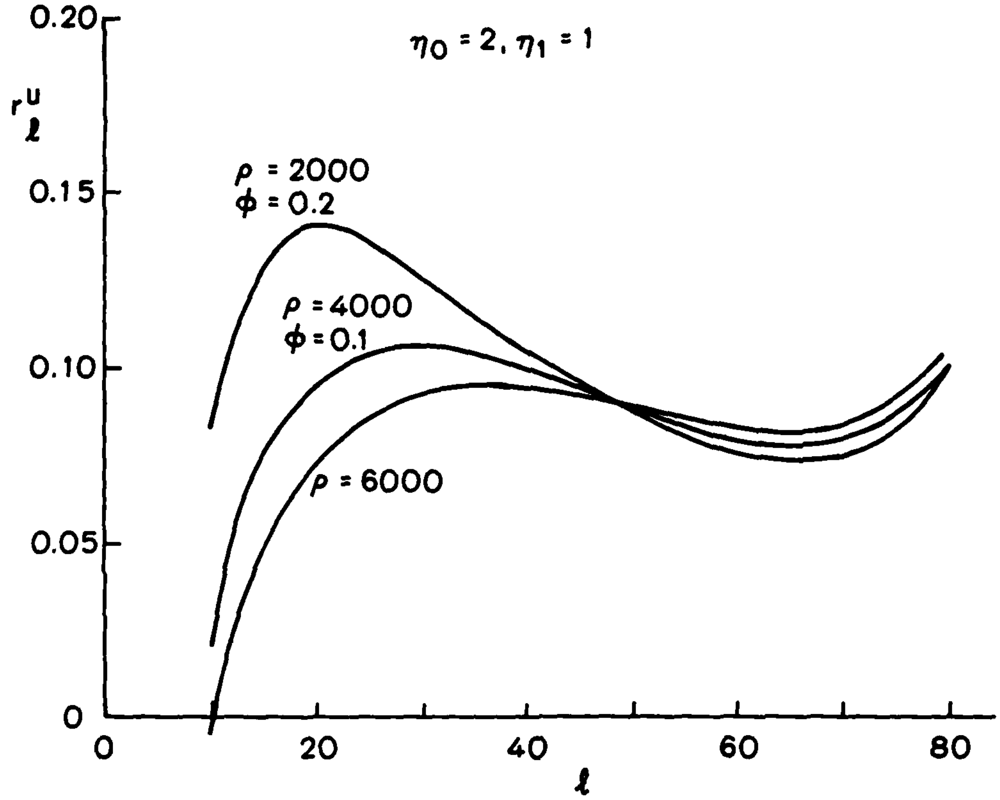

Figure 18.2 (d): Risk Aversion on Lifetime

shows risk aversion everywhere when $\eta_0 = 1$ and exhibits increasing risk aversion when $\eta_1 = 2$, but decreasing risk aversion when $\eta_1 = 1$. If the individual wishes to have his preferences described by such parameters, then Table 18.1 shows that v_s will be near 3.5 million dollars and p_{max} near 0.06.

19. Extensions

Our investigation has revealed several ways in which the basic model can be extended, but several other possibilities remain. An important extension would be to include a measure of health status to augment the present consumption and remaining life variables. While there is no generally accepted measure, one can construct a practical scale based on an individual's capability to participate in various life activities. Being able to incorporate a health indicator would permit extending the usefulness of the model to medical decisions where outcomes such as paralysis or limited activity were a possible consequence of the decision, or to the purchase of disability insurance.

While we have used a single consumption variable for simplicity, it should be clear that the formulation could be extended to year by year consumption measures, with joint probabilities assessed on the trajectory of annual consumption. A methodology for such probability assessments is presented by Buede [12]. This extension could be combined with the health status measure to provide a very complete, if difficult to assess, value function for an individual.

A final extension would be to the case of multiple deaths, which we call catastrophes. The perspective we have taken is the individual's - whether he is considering risks to his own life or

buying safety for someone else he values. This perspective is different from one that foresees a government role in collective safety decisions. A study by Ferreira and Slesin [13] suggests that society may be overspending in the prevention of catastrophes. To put it another way, the individual might well prefer that the funds devoted to disaster prevention and mitigation be spent in ways that reduced his probability of death, regardless of whether others were involved. Collective decisions can result in resource allocations that are not in the interest of any individual. However, a complete treatment of this issue will require more research.

20. Conclusions

We have presented a conceptual framework and mathematical formulation for decisions involving risks to life. The ethical basis for this investigation is that every individual has the right to make safety decisions regarding his own life.

The general model we have developed resolves a question that often troubles those who are concerned about "valuing" human life. They ask, "If you place a value on someone's life, does that mean that I can buy his life for that amount of money?" While the question is easy to answer "no" on ethical grounds, the answer that some simplistic schemes for life valuation would have to supply if they are to be consistent is "yes". <u>However, the formulation we have discussed shows that there is no inconsistency between refusing to sell your life for money and being willing to trade risk of death for money in small risk situations.</u> <u>The small risk value of life that we have determined is not a selling price for life and yet is a very practical basis for making the many decisions involving buying and selling hazards to life that we face over a lifetime.</u>

We have emphasized the use of numerical examples to provide a sense of magnitude to the results. The model we have used accounts for the major factors influencing a hazardous decision and yet is simple enough to place only reasonable demands on the individual who wishes to use it. We have avoided any examination of an "average" individual because we feel that

such results might be used to promote the idea of a standard small-risk life value, an idea that we believe is not in keeping with the ethical basis for the model. Nevertheless, it is quite clear from the numerous results and a limited amount of interviewing of people on these issues that many individuals feel comfortable with the idea of using their small-risk value in decision-making. Furthermore, in all cases, this small-risk value is many times the economic value that would be implied by commonly-used life valuation methods.

If these results hold generally, there are major implications for many decisions in our society. Each individual can use the small-risk value of his life in his own decision-making to assure that he will be consistent with his own preferences for risks affecting the quality and quantity of life. Furthermore, he would be wise to insist that his agents do likewise.

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No assertion can command attention in time of emergency like, "It's a matter of life and death". The problem of making decisions that can affect the likelihood of death is one of the most perplexing facing the analyst. As individuals, we are often called upon to make decisions that affect our safety, and others are increasingly making those decisions on our behalf. Yet most present approaches to life and death-decision making concentrate →

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on the value of an individual's life to others rather than to himself. These approaches are both technically and ethically questionable.

In this report, we develop a model for an individual who wishes to make life and death decisions on his own behalf or who wishes to delegate them to his agents. We show that an individual can use this model if he is willing to trade between the quality and the quantity of his life. A simplified version requires him to establish preference between the resources he disposes during his lifetime and the length of it, to establish probability assessments on these quantities, to characterize his ability to turn present cash into future income, and to specify his risk attitude. We can use this model to determine both what an individual would have to be paid to assume a given risk and what he would pay to avoid a given risk. The risks may range from those that are virtually infinitesimal to those that are imminently life threatening. We show that this model resolves a paradox posed by previously proposed models. In this model there is no inconsistency between an individual's refusing any amount of money, however large, to incur a large enough risk, and yet being willing to pay only a finite amount, his current wealth, to avoid certain death.

We find that in the normal range of safety decisions, say 10^{-3} or less probability of death, the individual has a small-risk value of life that he may use in the expected value sense for making safety decisions. This small-risk life value applies both to risk increasing and risk decreasing decisions, and is of the order of a few million dollars in the cases we have measured. This small-risk value of life is typically many times the economic value of life that has been computed by other methods. To the extent such economic values are used in decisions affecting the individual, they result in life risks that are in excess of what he would willingly accept. Using the small-risk life value as a basis for compensation should allow most risk-imposing projects to proceed without violating anyone's right to be free from significant involuntarily imposed hazards.

The report demonstrates the use of the model to treat hazards that continue over many years, to determine the size of contributions to saving the lives of others, and to incorporate more precise specifications of consumption-lifetime preferences.

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